THE MATHEMATICS STUDENT

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Vol. XXII

1954

PUBLISHED BY
THE INDIAN MATHEMATICAL SOCIETY

PRINTED AT THE
COMMERCIAL PRINTING PRESS
BOMBAY

CONNEMARA PUBLIC LIBRARY
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REPORT OF THE NINETEENTH CONFERENCE OF THE INDIAN MATHEMATICAL SOCIETY

MAY 1954

THE INDIAN MATHEMATICAL SOCIETY

NINETEENTH CONFERENCE, DELHI 1953

LEFT TO RIGHT

Sitting on the ground : Miss V. Thakurdas, Miss H. S. Katyayani, Miss Bhagya Lakshmi, Mrs. K. N. Kamalamma, Miss Girja Khanna, Mrs. Sreerangamma.

Sitting on chairs : S. Kerawala, S. M. Shah, V. Raghavachari, B. S. Madhava Rao, D. D. Kapadia, K. R. Gunjikar, R. Vaidyanathaswamy, G. S. Mahajani, Ram Behari, K. S. Krishnan, T. Vijayaraghavan, S. Mahadevan, B. N. Prasad, K. Chandrasekharan, S. Eilenberg, A. N. Singh, G. L. Chandratreya, D. S. Kothari.

Standing 1st row : K. Aggarwal, Hari Shankar, K. Rizvi, P. L. Bhatnagar, Gopi Chand, M. L. Puri, Parmanand, Hukam Chand, Prem Kumar Singhal, T. Natarajan, V. Seetharaman, Vencobarao, D. K. Sen, Omar Ali Siddiqi, K. V. Munuswami Reddy, N. Rama Rao, K. Raghavan Rao, S. Subramaniam.

Standing 2nd row : G. S. Sehgal, H. C. S. Sharma, O. P. Mital, D. R. Kaprekar, S. K. Abhyankar, S. K. Singh, D. N. Huddar, V. B. Cholker, Narendra Kumar, B. L. Sood, Jagannath Mittal, R. R. Sharma, K. Seetha Ram, S. S. Subramanyam, S. S. Lal Mathur, F. C. Auluck, R. S. Varma, Shri Krishna, Hansraj Gupta.

Standing 3rd row : Amolak Chand, Subba Rao, A. R. Rao, G. C. Patni, G. L. Setya, V. S. Huzurbazar, R. S. Mishra, G. T. Narayana Rao, Y. V. Thosar, B. L. Marwaha, Mukand Lal, M. Venkataraman, B. Ramamurti, P. D. Sukla, B. P. Gupta, B. K. Bhalla, S. N. Kevalgikar

Standing 4th row : A. N. Mehra, Ram Pershad, P. C. Mathur, L. C. Gupta, M. N. Khatri, K. P. Bhatnagar, D. R. Jain, V. P. Goyal, M. K. Singal, J. N. Kapur, S. D. Chopra, Hans Raj Aggarwal, M. S. Srinivasan, Bhagwan Das, Shanti Narayan, D. N. Abrol

Standing 5th row : B. S. Grewal, Ramanujam, K. Chimi Krishne, H. H. Thanawala, S. V. R. Naidu, C. N. Srinivasiengar, P. Kesava Menon, T. P. Srinivasan, M. S. Cheema, K. M. Shah, D. Chandra

MADRAS

PROGRAMME

Thursday, December 24, 1953

3.00 P.M.	Address of Welcome by Dr. G. S. Mahajani, Vice-Chancellor, University of Delhi Inauguration of the Conference by Dr. S. S. Bhatnagar, Secretary to the Ministry of Natural Resources and Scientific Research Opening of the Mathematical Exhibition by Dr. K. S. Krishnan, Director, National Physical Laboratory Annual Report by the Secretary of the Society, Professor S. Mahadevan Presidential Address by Professor Ram Behari
4.15 P.M.	Group Photograph
4.30 P.M.	At Home by the Reception Committee
5.15 P.M.	Popular Lecture by Dr. K. S. Krishnan on 'Geometrical Forms in Nature'
6.30 P.M.	Variety Entertainment

Friday, December 25, 1953

10.00 A.M.	Session for the presentation of papers
11.30 A.M.	Address by Dr. K. S. Krishnan on 'Brillouin Zones in Crystals'
12.00 NOON	Invited Address by Professor S. Eilenberg on 'Homotopy'
2.15 P.M.	Meeting of the Council
2.30 P.M.	General Meeting
3.00 P.M.	Symposium on 'Differential Geometry': Professor Ram Behari, Professor C. N. Srinivasaiengar, Professor V. S. Huzurbazar, Dr. R. S. Mishra, Dr. Sri Krishna, Mr. M. K. Singal
4.30 P.M.	At Home by the University of Delhi

5.15 P.M.	Popular Lecture by Professor D. S. Kothari on 'The problem of measurement in Quantum Mechanics'
6.30 P.M.	Musical Entertainment

Saturday, December 26, 1953

10.00 A.M.	Session for the presentation of papers
12.00 NOON	Invited Address by Professor W. Ambrose on 'Fibre Bundles and Differential Geometry'
2.00 P.M.	Symposium on Astro-Physics : Professor D. S. Kothari, Dr. P. L. Bhatnagar, Dr. M. K. Vainu Bappu, Dr. R. S. Kushwaha, Dr. A. M. Naqvi
3.30 P.M.	Visit to the National Physical Laboratory
5.00 P.M.	At Home by Dr. K. S. Krishnan, Director, National Physical Laboratory
6.00 P.M.	Popular Lecture by Dr. T. Vijayaraghavan on 'Mathematics and the Layman'

REPORT OF THE CONFERENCE

Inauguration

The Nineteenth Conference of the Indian Mathematical Society was held under the auspices of the University of Delhi, Thursday through Saturday, December 24-26, 1953. It was attended by more than 100 delegates from all parts of India.

The Conference opened on the afternoon of December 24, in the New Convocation Hall of the University of Delhi, with an address of welcome by Dr. G. S. Mahajani, Vice-Chancellor of the University and Chairman of the Reception Committee. It was then formally inaugurated by Dr. S. S. Bhatnagar, Secretary to the Ministry of Natural Resources and Scientific Research of the Government of India. In his inaugural speech Dr. Bhatnagar referred to the important contributions made by Indian mathematicians in ancient times, and to the more recent achievements of S. Ramanujan which have helped India again to find a place in the modern world of mathematics. "I am glad to say that after all these years it has been decided to perpetuate his memory by setting up the Ramanujan Institute. The credit for this development is in no small measure due to my friend Dr. Rm. Alagappa Chettiar, and we in the Council of Scientific and Industrial Research are happy to have contributed to it by providing a temporary habitat in our Central Electro-Chemical Research Institute at Karaikudi, till it is able to find a permanent home for itself. Government has provided finances for the foundation of a Ramanujan Chair of Mathematics. I am sure that this development will revive and stimulate a still stronger interest in mathematical research." Advanced mathematics now played a fundamental role in guaging the trend of events and fore-casting future developments, and formed the basis of all the engineering sciences, and should therefore be cultivated in the country. Dr. K. S. Krishnan, Director of the National Physical Laboratory, made a brief speech declaring the Mathematical Exhibition open.

Report for the year 1952-53

Professor S. Mahadevan, Secretary of the Society, then presented the report for 1952-53. He thanked the authorities of the University of Delhi for extending an invitation to the Society to meet in Delhi. The Society had met in Delhi twice before, in 1935 and 1945, and was glad to meet again in the capital of independent India. The fact that the Vice-Chancellor of the University of Delhi, Dr. G. S. Mahajani, and the President of the Conference, Dr. Ram Behari, were senior members of the Society was a matter of particular gratification. The Secretary said in his report :

" During the year under review the cruel hands of death have snatched from us two of our members, Prof. J. D. Newburgh of Tulane University, U.S.A. and Prof. A. A. Krishnaswamy Ayyangar, retired Professor of the University of Mysore. We offer our condolences to their bereaved families.

The activities of our Society during all these years have consisted in (i) the maintenance of a library containing some books and a large number of current and back numbers of important periodicals, (ii) the publication of two periodicals, both quarterlies, the *Journal of the Indian Mathematical Society* and the *Mathematics Student*, and (iii) the holding of periodical conferences to enable mathematicians to meet and discuss the latest advances in the subject. These conferences have also enabled us to discuss methods of improving the teaching of mathematics and of promoting research.

Our library is now located in the Ramanujan Institute at Madras and is looked after by Dr. Vijayaraghavan, the Director of the Institute. A Committee has been appointed to improve the book section of the library. The circulation of current journals is one of our important services to active mathematicians. This circulation has become curtailed owing to the dislocations due to the transfer of the library from Poona, but we are most anxious that our members should avail themselves of these services which are again functioning normally. We are planning the preparation of a complete

catalogue of our periodicals, in which there are many long runs with unfortunately some gaps here and there.

Of the two quarterlies, the *Journal of the Indian Mathematical Society* which is nearly fortyfour years old and is running through Volume XVII, second series, has steadily improved both in the standard of research papers published and its printing, thanks largely to the help rendered by the Tata Institute of Fundamental Research and the careful attention bestowed on it by the Editor, Professor K. Chandrasekharan. He took charge of the Journal at the beginning of 1950 and during the last four years, 139 papers were received for publication of which 53 were rejected, 58 accepted for publication and 28 are still under consideration. The referees were chosen from all over the world and were acknowledged experts in the concerned subjects.

The other quarterly, the *Mathematics Student*, which meets the needs of collegiate teachers and young research workers is over 20 years old. Profiting by the experience afforded by the transfer of the *Journal* to the Commercial Printing Press, Bombay, the Council decided to transfer the printing of the Student also to Bombay from January this year, the Editorship having been entrusted to Professor K. Chandrasekharan to improve the tone and the standard of publication. This action of the Council has been amply justified as a casual look at its latest issue will convince any impartial observer. The co-operation of all members is requested in the task of making the *Student* an up-to-date and regular bulletin of the activities of the Society and of its members.

We may be allowed to make the claim that the Society's periodicals now conform to established international standards.

In spite of what has been done so far, we have no reason for self-congratulation. For a country of our size and with our past contributions to ancient mathematics, our present research output is miserably poor, and cannot compare favourably even with the

output and scholarship of a single centre of learning like Göttingen of pre-war Germany. The scholarship of our college professors, even in the bigger cities, in modern mathematics is not up to date, and there is hardly any steady and genuine atmosphere of research even in our university centres. These are matters which should make us think furiously. Our university courses will have to be revised and rearranged from time to time so as to incorporate the clearer outlook given by developments in modern algebra and topology and to give a place for modern theories of integration in our college syllabus. A ray of hope in this gloom is the establishment of the Tata Institute of Fundamental Research in Bombay and of the Ramanujan Institute of Mathematics at Madras and the interest which the Government of India is now showing in the development of these Institutions.

Speaking of the Ramanujan Institute reminds us of Ramanujan whose famous notebooks still remain unpublished. It is true that some portions of these notebooks have been the subject of various papers; all the same there is a need for editing the remaining portions and publishing the material for the benefit of research workers in this field.

India is a large country and besides the annual conferences there is room for regional gatherings meeting more frequently and more informally and with a greater amount of time to devote to mathematical discussions. We are happy to note that the Annamalai University has taken an initiative in this direction, and that there is an active Mathematics Club in Madras. The Tata Institute of Fundamental Research is also acting as the Princeton of India by inviting talented men from abroad to come and lecture under its auspices. We hope that the Ramanujan Institute will follow suit and that facilities will also be provided for interested workers to attend these courses of lectures.

In this connection we welcome the appointment by the Central Government of a National Committee for Mathematics with Professor Bhabha as Chairman, Professor Chandrasekharan as Convener and

Secretary, and Professors Minakshisundaram and Ram Behari as members. We understand that it is the function of the Committee to keep a liaison with the newly created International Mathematical Union. The Union is seeking to organise symposia and conferences in various parts of the world both on the education and research levels, and we understand that efforts are being made to arrange an International Conference in India. The Government of India deserves to be congratulated on their decision to appoint a National Committee for Mathematics in India which we believe is a sign of recognitition of the importance of this basic science in our national culture.

Our finances are just sufficient at present to keep us going, but not for improving our library, or giving us a habitation of our own, or for the publication of monographs embodying work done in India. We request our members to pay their annual fees promptly and trust that the Government of India will come to our help. We have in this connection to thank the Universities of Madras, Bombay and Osmania, the Tata Institute of Fundamental Research and the National Institute of Sciences, for the generous grants they have been making from year to year."

Presidential Address

The President of the Society, Dr. Ram Behari, then delivered his address which is printed separately.

Business Meeting

A meeting of the Council of the Society was held on the afternoon of December 25, followed by a business meeting of the Society at which a resolution touching the death of Prof. A. A. Krishnaswami Ayyangar and Prof. J. D. Newburgh was passed.

Proceedings of the Conference

There were two sessions for the presentation of papers, one on December 25, and the other on December 26. Abstracts of these papers are printed separately. An invited address on Homotopy was given by Professor S. Eilenberg on December 24. Professor K. Chandrasekharan presided. Another address by Professor

W. Ambrose on Fibre Bundles and Differential Geometry was given on December 26. Two symposia were held, one on Differential Geometry with Professor Ram Behari as Chairman and Professor C. N. Srinivasiengar, Professor V. S. Huzurbazar, Dr. R. S. Mishra, Dr. Sri Krishna, and Mr. M. K. Singal as participants, and the other on Astro-Physics with Professor D. S. Kothari as Chairman and Dr. P. L. Bhatnagar, Dr. M. K. Vainu Bappu, Dr. R. S. Kushwaha and Dr. A. M. Naqvi as participants. Owing to shortage of time Dr. F. C. Auluck and Mr. Pyare Lal could not present their papers in the symposium. Dr. Gagan Behari Bandopadhyay who had sent the synopsis of a talk for the symposium could not attend it. There were three popular lectures : Dr. K. S. Krishnan spoke on Geometrical Forms in Nature on the first day of the Conference, Professor D. S. Kothari spoke on the Problem of Measurement in Quantum Mechanics on the second day, and Professor T. Vijayaraghavan spoke on Mathematics and the Layman, on the third day of the Conference. Professor Vijayaraghavan's lecture was the last item on the regular programme, at the end of which Professor Mahadevan expressed the sincere appreciation of all the delegates to the University of Delhi and to the Reception Committee for their warm hospitality.

Social Programme

The entertainment of the delegates included receptions and variety shows. The delegates were housed and meals served in Gwyer Hall. The Reception Committee was at home to the delegates on the afternoon of December 24 and a variety entertainment was arranged later that evening. The University of Delhi was at home to the delegates on the evening of December 25, and an elaborate musical entertainment was arranged later that evening. A visit to the National Physical Laboratory was arranged on the afternoon of December 26. The Director of the Laboratory, Dr. K. S. Krishnan, was at home to the delegates.

An exhibition of mathematical models and charts was arranged by the Reception Committee. A list of the exhibits is given separately.

PRESIDENTIAL ADDRESS

By RAM BEHARI

I am deeply conscious of the great honour which my colleagues in the Indian Mathematical Society have done me by electing me President of the Society for the ensuing term. My first duty on this occasion is to offer my grateful thanks to them all. While conscious of my own limitations, I have accepted this high office as it will give me an opportunity to serve the Society in yet another capacity. The Indian Mathematical Society was founded in the year 1907 by the late V. Ramaswami Aiyar for the advancement of higher mathematics and research. Its progress and development as an All-India organization is, I believe, essentially linked up with the scientific and industrial progress of our country, on which depends the welfare and prosperity of our nation. We are once more gathered here in this capital of our country for an exchange of ideas and reports on advances in mathematics, on the invitation of the University of Delhi through its Vice-Chancellor, Dr. G. S. Mahajani, who is himself a distinguished mathematician. Our sincere thanks are first due to our hosts. By inviting the Indian Mathematical Society to hold this Conference here they have shown a true understanding of the needs of science and the importance of mathematics for scientific development. To promote scientific progress in a country, it is not enough to encourage teaching and research in only those subjects that attract public attention on account of their obvious and immediate usefulness, but the importance of basic subjects like mathematics must also be recognised and encouraged. In Chicago in 1933-1934 was displayed the tree of knowledge. The branches of the tree were shown as Economics, Sociology, Medicine, Engineering, Forestry, Animal Husbandry, Public Health etc. The roots were shown as Physics, Chemistry, Botany, Zoology, Geology, Psychology, Bacteriology, Anthropology, Astronomy, etc. and the tap root was shown as Mathematics. No plant can grow without tap root ; so also the applied sciences, which are like the flower and fruit of the plant,

cannot flourish unless the basic subject of mathematics develops vigorously.

Sir J. J. Thomson, once pleading for the claims of pure research, which were in some danger of being swamped by those of applied research, asserted that "*Pure science is the seed of applied science and to neglect pure science in a thing of this kind would be like spending a very large amount on manuring and ploughing the land and then to omit the sowing of any kind*".

Regarding the importance of mathematics, I would like to repeat the words of Professor A. Narasinga Rao who, in his presidential address at the Fifteenth Conference of our Society, remarked :

"Mathematics represents perhaps the highest efflorescence of the creative human spirit, rivalling the fine arts in its aesthetic appeal, excelling poetry in its condensed richness, and akin to Upanishadic thought in its all-embracing abstractness. It is therefore a touch-stone of intellectual and moral maturity, and a nation otherwise sound but whose mathematics is not up to the mark is like a good-looking and well-dressed person without character. The surest way to commit national suicide is to neglect mathematics."

Mathematics has a great cultural value. The influence of mathematics on scientific development and through science on our daily lives and thoughts, on philosophy, religion, law, social sciences, etc. should be sufficient to establish the point that mathematics affects and to some extent determines our civilization. Keyser published in *Scripta Mathematica* a thesis that the type of mathematics found in any major culture is a clue or a key to the distinctive character of the culture taken as a whole. The history of mathematical thought is inter-related with the history of civilization.

For example, mathematics of the Greeks reveals the following characteristics. It was static. It was confined to bounded figures, and to geometrical objects which could be visualised. Negative numbers and irrational and imaginary numbers were unknown to them.

Emphasis was on the form in mathematics. Compare these characteristics with Greek life and thought. Greek architecture was static. They were unexplorative. They lacked perspective in painting. Form is a cultural notion in Greek philosophy and art. Let us look at the nineteenth century. Mathematics became dynamic with the completion of a satisfactory theory of irrational and complex numbers and with the rise of non-Euclidean geometries. The mathematicians constructed a logical theory about infinity, and began to study spaces which had no correspondence with the physical world. These characteristics of mathematics led to a different culture and civilization. Let us look at the twentieth century. The tendency towards more and more generalization and abstraction in mathematics is one of the most outstanding landmarks of the twentieth century. To illustrate this, just by one example, I shall take the case of geometry, and shall show how it has freed itself from the shackles of tradition by generalization and abstraction. Solid space for the Greeks had 3 dimensions, say, length, breadth and thickness. When Descartes invented analytical geometry by inventing the coordinates, geometry began to be studied algebraically or analytically instead of synthetically, i.e. the properties of geometrical figures came to be studied by their equations and not by their figures, the restriction to 3 dimensions was no longer necessary. First, in analytical mechanics in the eighteenth century it became useful to reason about solid space and time together as a geometry of 4 dimensions. Cayley in 1843 took the step from 4 to n where n was any, whole, positive, finite number. In 1906 Hilbert took a long stride from n dimensions to a *countable infinity* of dimensions, i.e. as many as there are of all the positive whole numbers 1, 2, 3, etc. The next step from a countable infinity of dimensions to an *uncountable infinity* of dimensions was taken about 1920. These look inconceivable yet are supported by the two most outstanding discoveries of the twentieth century, viz. the theory of relativity and quantum mechanics, for the theory of relativity is based on a particular geometry of 4 dimensions and the mechanics of atoms uses geometry of an infinity of dimensions.

This abstraction has affected civilization a great deal. People have been furnished with a tool which enables them to become more and more explorative in their attempts to harness nature.

One of the functions of our Society is to hold conferences like this so that our members may get a chance to exchange ideas by personal contact and collaboration. Though any mathematical act either of creation or apprehension, takes place in the deepest recesses of the individual mind, nevertheless mathematical thoughts must be communicated to others and assimilated in the body of general knowledge. Otherwise they can hardly be said to exist. It is necessary for mathematicians to group themselves together as mathematicians and to feel connected as members of a big family of common interest. Our common enterprises, journals, other publications and conferences are therefore of great importance. We who are gathered here represent the mathematical community of our country and represent most of the present-day currents of mathematical thought. This Conference will fulfil its function if by its opportunities for discussion and the exchange of opinions, it provides the stimulus to us, teachers, to do our duty to our pupils, and if it proves to be a source of encouragement and inspiration in mathematical research to all our members and others who join in our proceedings.

I presume that it is not inappropriate in a conference like this which gives the only opportunity for so many teachers and research workers to get together for exchange of ideas, to devote a little time to a consideration of mathematics as a whole. The technical papers and symposia which we shall have from tomorrow in the morning and afternoon sessions will deal with special fields or branches of mathematics. I shall take this opportunity to consider some general factors which affect the progress and development of mathematics in our country. The technical part of my address will be given under the symposium on differential geometry which has been fixed for tomorrow.

The average man's attitude towards mathematics is determined by the application of mathematics to physical and engineering

sciences with their wonderful achievements contributing so much to the development of civilization, and to the precision and infallibility of mathematical calculations. But mathematics is not a child of the age of engineering. Mathematics existed even when there were no telegraph lines, railways, motor cars, steamers, aeroplanes, etc. The average man, however, will be surprised to be told that mathematicians have quite a different idea about their own business. A mathematician is not concerned with the applicability or otherwise of his results. The theory of conic sections was developed by the Greeks 2000 years before it was applied by Kepler to astronomy and navigation. Maxwell's electrical theory was developed by mathematical methods in the middle of the nineteenth century without realizing that fifty years later, it would lead to the development of wireless telegraphy and radio—and later to radar. Newton and Leibniz discovered the calculus without realising the numerous applications which would result from it. Cauchy never dreamt that his theory of functions of a complex variable would one day be useful in the calculations of technical electricity and in the construction of electronic machines. The complicated theory of tensors was established by Riemann, Christoffel, Ricci, and Levi Civita long before it was used by Einstein as the backbone of the general theory of relativity. Abstract algebra as it has developed since 1910 was developed without any one of the algebraists concerned suspecting or foreseeing that it would find use in the physical sciences particularly in modern quantum mechanics. A mathematician works for the intellectual joy of creation. He enjoys the harmonious beauty of his subject and the triumphant moments of successful thinking. To give one or two instances of this attitude, I may mention Sylvester who, when asked whether the subject of his lecture namely '*substitution groups*' had any practical use, replied "Thank God so far as I know it has not any". A Cambridge don while proposing a toast of pure mathematics is said to have remarked "May it never find any use". A mathematician creates ideal situations and builds up coherent and consistent systems. They may be real or imaginary. For instance, he may imagine a

world where fruits, instead of falling down from trees go upwards, where the law of force is the inverse cube or the nth power of the distance instead of the inverse square. All that he is concerned with is internal consistency.

Besides providing intellectual joy of creation and application to physical and social sciences, mathematics is important for the sake of its bearings. The study of mathematics has a great cultural and educative value, since it inculcates regard for truth, intellectual honesty, exactness, clarity and precision. Mathematics seems to have been wrought into the very fibre of man's mind and its development and manifestation are in direct proportion to the degree of civilization which man attains. The kinds of mathematics developed by different peoples in scattered parts of the world all agree. Mathematical methods are the surest that men know and its results are the most dependable of any the world has ever found. In these days of specialisation, when there is more and more demand for the study of science and technology, the study of classics is being relegated to the background with the result that higher values of life are getting lost. Mathematics fills the need as it is one of the foremost disciplines for the acquisition of a liberal education. According to an old Sanskrit proverb "That is real education which leads to the liberation of the mind: all else is just training in craftsmanship." Classics and humanities have been studied as a discipline, as a liberal education; science also is a discipline, but mathematics being both an arts subject and a science subject is an ideal discipline.

Mathematics is the basis of all the science subjects, with which it is connected in an indissoluble manner. All natural sciences use mathematics in the formulation of their abstract theories. A mathematician is concerned not with the solution of particular problems in natural sciences but with the principle of natural sciences in general; he goes to the essence of the problem and tries to find a general solution which will equally apply to all fields. Mathematics supplies a language for the treatment of the quantitative problems of the physical sciences in the form of symbols.

It supplies sciences with ideas and methods with which to describe phenomena, for example, idea of a functional relation, statistical law which enables scientists to reason about situations which appear to be utterly chaotic. It prepares the minds of men for new ways of thinking, for example, discovery of the systems of non-Euclidean geometries, change in the conception of space brought about by Minkowski and Einstein; it enables the sciences to predict conclusions, e.g. deviations in the observed motion of the planet Uranus enabled Adams and Leverrier to predict the existence and the exact position in the sky of the unknown planet Neptune.

Mathematics which Gauss called the queen of the sciences is also one of the great arts. A mathematician, besides being a logician, is also an artist, who expresses himself with that precision, economy and conciseness which form some of the distinguishing qualities of art. The great philosopher Kant once remarked that the degree of development of a science depends upon the extent to which it has become mathematical. Sooner or later every true science tends to become mathematical. Mathematical method is, in essence, one and the same whether employed in physical sciences or in social sciences. There is no field in which a worker can avoid the mathematical method. Mathematical procedure furnishes a model for the guidance of all discourse of reason, no matter what the subject or field to which the discourse pertains. As mathematics is the basis of pure science, it is essential for our progress in the long run that we should devote our resources to it in proportion to its importance and should train a large number of our young mathematicians for careers of active research. This should be done as a measure of long-planning, in the same spirit in which the National Physical and Chemical Laboratories are being organised. It is hoped that Dr. Shanti Swarup Bhatnagar who is the one man who has been the motive force in bringing into existence a chain of National Laboratories in our country will be pleased to explore the feasibility of installing a National Institute of Mathematics where research in both pure and applied mathematics will be undertaken to meet the increasing demand for mathematics in most of the physical

and applied sciences. This will literally implement the suggestion made by him in his convocation address at the Baroda University, viz. "The scope of research work may be likened to a continuous spectrum at one end of which is pure academic work of the highest quality and at the other the technical development of processes and equipment."

Our country had a glorious past and a great civilization ; it should be noted that our mathematics was a true index of our greatness. Mathematics occupied a very high place in life in India in ancient times. It was considered to be the most superior of all the arts and sciences. The fourth verse of *Vedanga Jyotisa* reads as follows :

यथा शिखा मयूराणां, नागानां मणयो यथा ॥

तद्वद् वेदाङ्ग शास्त्राणां, गणितं मूर्धनि स्थितम् ॥ १ ॥

" As are the crests on the heads of peacocks, as are the gems on the heads of the snakes, so is the *ganit* at the top of the sciences known as the *Vedanga.* " The whole world acknowledges the debt of gratitude to our country for its pioneering achievements in systematic mathematics. There is the discovery of the decimal place value system of notation, which, of all mathematical inventions, has contributed most to the progress of science and civilization, and without which modern mathematical science could not have made as much progress as it has made to-day. Then there is the invention of zero, which Professor Schrodinger calls in his book *Space-Time-Structure*, "the most important number in mathematics." About the significance of this discovery Professor Halsted said, " this giving to airy nothing not merely a local habitation and a name, a picture, a symbol, but helpful power is the characteristic of the Hindu race whence it sprang. It is like coining the Nirwana into Dynamos. No single mathematical creation has been more potent for the general on-go of intelligence and power ". Furthermore there are the generalizations of algebra, the sine function and the series now known as Gregory's series in trigonometry, and the foundations of indeterminate analysis. India may claim to be the birth place of arithmetic and

algebra. Instances are found in the works of some of the ancient Indian mathematicians, Aryabhatta, Varaha Mihira, Brahmagupta, Sridhara, Mahavira and Bhaskaracharya which show that they anticipated the work which engaged the attention of European masterminds 500 years later. India was a leading country of the world upto the fifteenth or sixteenth century. It is unfortunate that after the time of Bhaskara about 1150 A.D. very little original work was done in India on account of various political and other causes. Indian mathematics migrated to Europe through the Arabs *via* Spain. This knowledge and this experience had in them the origin of the forces out of which grew the modern European mathematics.

At the beginning of the twentieth century mathematical activity along modern lines began to spring up in India under the inspiration of Indian students who learnt mathematics abroad. The history of modern mathematical work in India is not even about 50 years old. Indian contribution in this period, however, is quite appreciable. India has produced in this period Srinivasa Ramanujan (1887-1920) who though born in a poor family, educated only upto the Intermediate standard, and employed as a petty clerk on Rs. 30/- a month in a local office in Madras made such outstanding contribution to mathematics in the fields of theory of numbers and theory of elliptic and modular functions that he became a Fellow of Trinity College, Cambridge, and a Fellow of the Royal Society at the age of 31. Professor G. H. Hardy, one of the greatest mathematicians of this century, while paying a tribute to him and his work wrote, "his work has one gift which no one can deny, profound and invincible originality. On this side most certainly I have never met his equal, and I can only compare him with Euler and Jacobi. European mathematicians will take fifty years to decipher what is contained in his notebooks ". His early death at the age of about 33 in 1920, was the greatest mishap to Indian mathematics in recent times. However, this solitary example has shown that the continuity of the genius of the Indian people exists and that suitable opportunities being given, Indians will restore to India her pristine honoured role of intellectual leadership, contributing their fullest quota to the fund of mathe-

matical knowledge. It is the duty of the mathematicians of our country to achieve something worthy of our illustrious past.

At a conference like this our Society should take stock of the position of teaching and research in mathematics in the country and should suggest necessary steps for improvement. We have in India at present a vast reservoir of talent and with proper opportunities and facilities to research workers our achievements can be as glorious as they have been in ancient times. It is gratifying to find that in different parts of our country some new branches of mathematics are being cultivated, and new tendencies in research are showing themselves with the result that modern mathematics is getting more and more assimilated. India is a member of the International Mathematical Union. Some young Indian mathematicians both in India and abroad are doing work of very high order. Besides research work that is being done in our universities there are three institutes for pursuing research in pure science, namely, (1) Tata Institute of Fundamental Research in Bombay (2) The Ramanujan Institute of Mathematics at Madras (3) Prof. C. V. Raman's Institute at Bangalore. All this shows that we are not stationary. But at the same time it has to be confessed that as compared to the size and population of our big country the output of original work is not enough and that we are still far from the ideal namely the creative assimilation and output in all the branches of mathematics. In our universities we are not yet studying all the modern subjects that are engaging the attention of European and American mathematicians. For example, at the University of Cambridge, England, for the mathematical tripos examination regular courses of lectures are delivered on various branches like Linear Algebra, Abstract Algebra, Algebraic and Analytic Topology, Geometry of Numbers, Finite Groups, Theory of Aggregates and Transfinite Numbers, n-Dimensional Geometry, Modular Functions, Integral Functions, Functions of Mathematical Physics, Homotopy, Absolute and Non-Absolute Integration, Numerical solution of Differential Equations, Thermodynamics, Statistical Mechanics and Quantum Mechanics, Fluid Mechanics and Turbulence, Dynamical Meteorology,

Rotating Masses of Liquid, Stellar Structure, Random Variables, Stochastic processes, and Design of experiments. These are not taught in the mathematics departments of our universities to any great extent.

It is very desirable that our universities or the members of our Society in groups should take steps to study these branches so that the whole field of mathematics is cultivated in the country. The help of mathematics is required by all scientists, particularly theoretical physicists, statisticians, engineers, and defence services. Hence mathematicians have to play an important part in the huge task of national reconstruction. We should, therefore, examine the causes which are retarding the progress of mathematical research and should suggest remedies. In my opinion some of the causes of our lack of progress are as follows :

1. Curricula in most of our universities are antiquated. The curricula to-day are more or less the same as we used to have about 30 or 40 years ago. Considerable time and energy of students and teachers are therefore wasted on topics which have lost their utility, like Horner's method of solving numerical equations which can now be done by calculating machines, and in solving artificial and tricky problems, with the result that there is no time left for introducing modern branches. Some universities have introduced modern subjects in their post-graduate course but are not actually providing teaching in all these subjects on account of non-availability of suitable qualified staff or other adequate facilities.

2. The tragic element in the present situation is the amount of research talent that is wasted. Adequate number of research scholarships is not offered and there is no certainty of research scholars being absorbed in the teaching staff even after obtaining research degrees. Therefore they do not apply themselves wholeheartedly to their research work and keep trying for openings in other walks of life.

3. The scales of pay of the junior teachers in our colleges and universities are so low that it has become difficult to attract good

workers and to retain those who are already there, since they get more remunerative jobs in other fields. Teachers are expected to possess high ideals and to devote all their time and energy to teaching and research but they are paid such low wages that it is impossible for them to keep themselves intact and to provide their families the bare necessities of life.

4. The value of mathematics and the value of mathematical ability to do research are not recognised to the extent to which they should be. Mathematics teachers are hampered by being put on to unacademic work like preparing the timetables, date-sheets of examinations, calculation of percentage of attendance, tabulation of results, etc. and are denied the leisure required for the pursuit of creative work and extension of the boundaries of knowledge.

Difficulties like these must be remedied by adopting the following steps :

1. Teaching and research in universities must be improved. Special responsibility therefore rests on college and university teachers, for it is they who are expected to be wedded to mathematics, because they have got greater access to books and periodicals and have greater chances of exchange of ideas than those who are not directly engaged in such teaching. They should consider themselves fortunate in having chosen to devote their lives to a subject of so fundamental a character whose history goes back to the earliest period of which we have any record, which has almost unlimited applications and for which there is a growing demand in almost all types of human endeavour. Professor Sir Lawrence Bragg, Cavendish Professor of Experimental Physics, while speaking on Administration and Academic duties at the occasion of the Conference of Universities of the Commonwealth held at Cambridge in July, 1953, defined the functions of the teacher and research guide by saying that " in teaching, our function is to train our young people in good ' style ' of thinking just as a games expert teaches a young player how to make strokes in the right way. In addition we have to assess the students ' points ' just as a trainer assesses those of a race horse.

In research our main pre-occupation is to place the young research student in the right environment and to make him feel how tremendously interesting his research is. At the same time we must have time to think about his work and plan its strategy. The most important thing in research is the flash of imagination which initiates a new line of investigation. It is impossible to be sure the inspiration will come but one can define precisely the conditions under which it is certain not to come. These are pre-occupation with agenda and a full engagement book ". A mathematics teacher should try to discover the ability and aptitude for research in his students but the teacher will find it difficult to create love for research in his pupils if he has not gone through the experience of mathematical discovery himself. There should therefore be unity of teaching and research. University teaching cannot be separated from research except to its detriment and degradation. Seminars must be introduced where discussions on advanced subjects may be conducted by internal teachers and other invited experts from outside for exchange of ideas.

Modern subjects should be studied by our members in groups, based on the principle of study in close association. Members will do well to remember the words of the founder in this connection that in the study of any one field there are four stages of progress which are symbolised by the letters of the mathematical word '*Mile*', *e* denoting the stage of *an entrant*, *l* denoting the stage of a *learner*, *i* denoting the stage of an *interpreter*, when the essential things are taken hold of and the non-essential things which are like cobwebs in the mind are swept out, *M* denoting the stage of the *master*, when one brings a fresh light of one's own into the subject, becomes a discoverer and extends the scope of the field. It will be advantageous for us to write books on advanced subjects on the lines of the colloquium publications of the American Mathematical Society.

Diffusion and interaction of mathematical ideas through standardization of symbols, publications, and personal contacts, which have been responsible for tremendous progress in mathematics in the last thirty years, should be encouraged. Any step that retards

international contacts through the medium of publications, such as restriction to languages not widely read is an anti-mathematical act and will handicap one's own achievements.

2. The value of mathematical research should get high recognition and research workers who create mathematics should be considered as national assets. Teachers should be given leisure hours for seminars, study and contemplation. They should be actuated by a burning desire to restore their country to her ancient honoured role.

3. Regarding the salaries of teachers and their status in society, I cannot do better than quote from the speech of Dr. S. Radhakrishnan delivered at the Convocation of the University of Delhi, held on December 5, 1953, when he said, " if this country is to participate in the march of mind in science and scholarship, universities must recruit for their staff some of the best minds of the country. The university teacher should be helped to live in comfort, if he is to devote himself to learning, teaching and research. As the young recruits to the universities are to-day low salaried, they fail to appreciate intellectual values and get interested in writing text books or obtaining examinerships. I hope the university services will become as attractive as All-India services, for that is the only way to recruit and retain some of the ablest persons for the universities. As the example of the teacher has great influence among the pupils, we cannot evade our respect to the teaching profession ."

In conclusion, I beg to appeal to all those present here and others interested in mathematics to give their hearty cooperation to the Indian Mathematical Society in its efforts to promote higher mathematical study and research in India by helping it to get a fully equipped library and by improving the quality and quantity of its journals, so that our country may once again find a most prominent place on the mathematical map of the world and in the words of the founder of our Society, " that we may hold the banner of Mathematics aloft, as the motherland marches to glory with the rest of the world ".

SYMPOSIUM ON DIFFERENTIAL GEOMETRY

Chairman: PROFESSOR RAM BEHARI

Professor Ram Behari said in his opening remarks that considerable work in Differential Geometry is being done on embedding theory in metric, Weyl, affine, projective and conformal spaces, collineations in generalized spaces, the non-infinitesimal approach to problems of Differential Geometry, affine and projective geometries of systems of hypersurfaces, Finsler geometry, contact transformations, infinitesimal structures and certain spaces of Lie, special congruences, unified field theories, global properties of Riemann spaces and Finsler spaces, differentiable manifolds, fiber bundles, etc. He announced that five other speakers, Professor C. N. Srinivasienger, Professor V. S. Huzurbazar, Dr. R. S. Mishra, Dr. Shri Krishna and Mr. M. K. Singal would speak on different topics of interest and that he would confine himself to the generalized theory of gravitation. After giving the outlines of the general theory of relativity he explained Einstein's generalized theory of gravitation [*The Meaning of Relativity,* Princeton (1950), 127-141] leading to the field equations

$$R_{ik} = 0, \; g^{ik}_{+-;l} = 0, \; \Gamma_i = 0,$$

which Einstein describes as a system of 'utmost simplicity' and asserts that they appear to him 'the natural extension of the general theory of relativity'. He then described the contributions of Narlikar and Tiwari [*Proc. Nat. Inst. Sci.* 15 (1949), 73-79, 249-261; *Phys. Rev.* 76 (1949), 868], Kursunoglu [*Proc. Phys. Soc.* 65 (1952), 81-83], Papapetrou, [*Proc. Royal Irish Acad.* A, 52 (1948), 11; *Proc. Phys. Soc.* A, 64 (1951), 57-75, 302-310] Bandopadhyay [*Nature,* 167 (1951), 648], Schrödinger, [*Proc. Royal Irish Acad.* A, 51 (1947), 205-216; *Ibid.* 51 (1948), 163-171; 52 (1948), 1-9], and referred to the recent work of Professor S. N. Bose [*Comptes Rendus,* Paris 236 (1953), 1333-1335; *Le Jour. Phys. et le Radium,* 14 (1953), 641-644; 645-647], viz. his method of obtaining the divergence identities in Einstein's unified field theory by the use of a variational

principle without using the equations relating the affine connection $\Gamma^{\lambda}_{\mu\nu}$ to the $g_{\mu\nu}$ and his work on a unified field theory with $\Gamma_{\mu} \neq 0$ and on certain consequences of the existence of the tensor g in a relativistic affine field. He also referred to the work of S. N. Gupta [*Proc. Phys. Soc.* A, 65 (1952), 608-619] regarding the quantization of Einstein's gravitational field and to his investigations of the interaction of gravitational quanta with other elementary particles. He then mentioned Hlavaty's [*Proc. Nat. Acad. Sci. U.S.A.* 38 (1952), 243-247, 1052-1058. *Jour. Rat. Mech. Anal.*, 1 (1952), 539-562. *Ibid.* 2 (1953), 1-52. *Inter. Math. News* (1953) December, p. 3] tensorial solution of the equations in the basic problem, namely, to find the geometry implied by the 64 equations relating the affine connection to a fundamental quadratic tensor which is neither symmetric nor skew-symmetric, and remarked that this work may have important physical consequences.

In conclusion he pointed out that no solution of the field equations free of singularities has yet been found and that the search for a perfect unified field theory of gravitation and electromagnetic phenomena is still going on.

Professor C. N. Srinivasiengar spoke on the skewness of distribution of a generator of a scroll as defined by V. R. Chariar [*Bull. Calcutta Math. Soc.* 37 (1945), 133-136] and pointed out that skewness which is an invariant is not a fundamentally new invariant, for it can be expressed as

$$\gamma = -\frac{J_0}{\sqrt{(-K_0)}} - \cot\theta,$$

where J_0 and K_0 are respectively the mean curvature and the Gaussian curvature at the central point, and θ is the angle at which the line of striction cuts the generator. He then mentioned the following elegant results :

$$\frac{1}{\beta} = -(T_g + k_n \cot\theta),\ \gamma = \cot(\theta + \phi),$$

where $\tan\phi = k_n | T_g$, T_g and k_n are in the direction of the line of striction, and made some inferences therefrom.

He also referred to a paper by E. Kruppa [*Osterreich Akad. Wiss. Math. Nat.* KL, S-B, IIa, 157 (1950), 143-76] and mentioned the result that " If we construct a geodesic on a surface whose tangent at any point coincides with the direction of the generator through the point, the ratio of the torsion to the curvature of this curve is the value of the skewness of the corresponding generator."

Professor V. S. Huzurbazar, who spoke next, mentioned an interesting relationship between Riemannian Geometry and the theory of probability, which was first discovered by Sir H. Jeffreys [*Proc. Roy. Soc.* A. 186 (1946), 453] which helps in stating the prior probability of parameters in estimation problems. He also discussed the problem of finding orthogonal parameters for a given probability distribution pointing out difficulties and complexitics inherent in the problem [*Proc. Cambridge Phil. Soc.* 46 (1950), 281].

Dr. R. S. Mishra spoke on generalized Riemannian spaces. He gave a brief resumé of the results obtained by him on the sub-spaces. He also obtained generalizations of the generalized Mainardi Codazzi equations and stated that curvature vectors of a curve C in a generalized V_n and V_m are the same as those in a Riemannian V_n and V_m.

Dr. Sri Krishna spoke on the congruences of curves in a Riemannian space. He described the work that has been done on the subject by Springer and other workers including himself. He also showed that a congruence of curves in a V_n is capable of hyper-spherical representation only if the enveloping V_{n+1} is flat.

M. K. Singal spoke on totally geodesic sub-spaces and pointed out that (i) the normals to a totally geodesic sub-space of a Riemannian space are Ricci principal directions for the enveloping space at points of the sub-space and (ii) if a V_{n+1} admits ∞^1 totally geodesic hyper-surfaces V_n, the tangential directions at any point are Ricci principal directions for V_{n+1}, if and only if, all the hyper-surfaces are Einstein spaces. He also deduced certain other results for totally geodesic sub-spaces of a Riemannian space.

MAY 1954

GENERALIZATION OF CODAZZI'S EQUATIONS IN A SUB-SPACE IMBEDDED IN A RIEMANNIAN MANIFOLD

By M. K. SINGAL and RAM BEHARI

1. Introduction. In a joint paper, Behari and Mishra (1949) have obtained expressions for $\nu_a \equiv \chi^i . \lambda^i_{,a}$, where χ^i and λ^i are respectively the direction cosines of the normal to the surface at the point x^i and the ray of the congruence through x^i. It has also been shown that if the congruence be one of normals to the surface of reference, these equations reduce to Mainardi Codazzi equations. Mishra (1952) has obtained generalized expressions for these invariants in the case of a congruence of curves, one of which passes through each point of a hyper-surface V_n of a Riemannian manifold V_{n+1}. *It is proposed to obtain generalized expressions for a set of* $(m-n)$ *congruences of curves, one curve only of each of which passes through each point of a sub-space* V_n *of a Riemannian manifold* V_m. The corresponding results for a hyper-surface V_n of V_{n+1}, and as a particular case, Mainardi Codazzi relations in an Euclidean space of three dimensions, Mainardi Codazzi relations and condition of integrability for a sub-space V_n of V_m, Mainardi Codazzi relations for a hyper-surface V_n of V_{n+1}, and as a particular case for a surface in ordinary space, have been deduced.

2. Consider a sub-space V_n of co-ordinates x^i $(i = 1, 2, \ldots, n)$, and metric $ds^2 = g_{ij}\, dx^i\, dx^j$ imbedded in a Riemannian manifold V_m of co-ordinates y^α and metric

$$ds^2 = a_{\alpha\beta}\, dy^\alpha\, dy^\beta . \tag{2.1}$$

We have then the relation

$$a_{\alpha\beta}\, y^\alpha_{,i}\, y^\beta_{,j} = g_{ij}, \tag{2.2}$$

where (,) denotes covariant derivatives with respect to the x's.

Throughout the present investigation Latin indices take the values $1, 2, \ldots, n$ and Greek indices $\alpha, \beta, \gamma, \delta, \ldots, \rho$ take the values $1, 2, \ldots, m$ and Greek indices $\nu, \mu, \sigma, \tau,$ take the values $n+1$

$n+2,\ldots, m$. Let $N^{\alpha}_{\nu|}$ be the contravariant components in the y's of a system of $(m-n)$ unit vectors normal to V_n. These vectors satisfy the relations (Weatherburn 1950)

$$a_{\alpha\beta} N^{\alpha}_{\nu|} N^{\beta}_{\nu|} = 1, \tag{2.3}$$

$$a_{\alpha\beta} N^{\alpha}_{\mu|} N^{\beta}_{\nu|} = 0, \ (\mu \neq \nu), \tag{2.4}$$

$$a_{\alpha\beta} y^{\alpha}_{,i} N^{\beta}_{\nu|} = 0, \tag{2.5}$$

and

$$y^{\alpha}_{;ij} = \sum_{\nu} \Omega_{\nu|ij} N^{\alpha}_{\nu|}, \tag{2.6}$$

where (;) denotes generalized co-variant derivative or tensor derivative and $\Omega_{\nu|ij}$ are the components of a covariant tensor of the second rank symmetric in i and j and given by

$$\Omega_{\nu|ij} = y^{\alpha}_{;ij} a_{\alpha\beta} N^{\beta}_{\nu|}. \tag{2.7}$$

Also

$$N^{\alpha}_{\nu|;i} = -\Omega_{\nu|ik} g^{jk} y^{\alpha}_{,j} + \sum_{\mu} \vartheta_{\mu\nu|i} N^{\alpha}_{\mu|}, \tag{2.8}$$

where

$$\vartheta_{\mu\nu|i} = a_{\alpha\beta} N^{\alpha}_{\nu|;i} N^{\beta}_{\mu|}, \tag{2.9}$$

so that $\vartheta_{\mu\nu|i}$ is skew symmetric in μ and ν.

3. Let us consider a set of $(m-n)$ congruences of curves, one curve of each of which passes through each point of the sub-space V_n. Let $\lambda^{\alpha}_{\tau|}$ be the contravariant components in the y's of a unit vector in the direction of the curve of the congruence $\lambda_{\tau|}$. Since the vector is, in general, not normal to V_n, it may be expressed linearly in terms of $y^{\alpha}_{,i}$ and the set of normals $N^{\alpha}_{\nu|}$. Thus

$$\lambda^{\alpha}_{\tau|} = t^{i}_{\tau|} y^{\alpha}_{,i} + \sum c_{\nu\tau|} N^{\alpha}_{\nu|}, \tag{3.1}$$

where the parameters $t^{i}_{\tau|}$ and $c_{\nu\tau|}$ are such that *if* $\theta_{\nu\tau|}$ is the angle between the vectors $\lambda^{i}_{\tau|}$ and $N^{\alpha}_{\nu|}$, then

$$c_{\nu\tau|} = \cos\theta_{\nu\tau} = a_{\alpha\beta} \lambda^{\alpha}_{\tau|} N^{\beta}_{\nu|}, \tag{3.2}$$

and

$$1 - t^{i}_{\tau|} t_{\tau|i} = \sum \cos^2\theta_{\nu\tau|}, \tag{3.3}$$

for

$$a_{\alpha\beta} \lambda^{\alpha}_{\tau|} \lambda^{\beta}_{\tau|} = a_{\alpha\beta} (t^{i}_{\tau|} y^{\alpha}_{,i} + \sum_{\nu} c_{\nu\tau} N^{\alpha}_{\nu|}) (t^{j}_{\tau|} y^{\beta}_{,j} + \sum_{\nu} c_{\nu\tau} N^{\beta}_{\nu|})$$

or

$$1 = g_{ij} t^{i}_{\tau} t^{j}_{\tau|} + \sum_{\nu} c^2_{\nu\tau}.$$

Taking the generalized covariant derivative of (3.1) with respect to x^j, we get

$$\lambda^a_{\tau|;j} = t^i_{\tau|;j}\, y^a_{,i} + t^i_{\tau|}\, y^a_{;ij} + \Sigma\, c_{\nu\tau|}\, N^a_{\nu|;j} + \Sigma\, c_{\nu\tau;j}\, N^a_{\nu|}. \tag{3.4}$$

Making use of (2.6) and (2.8), (3.4) takes the form

$$\lambda^a_{\tau|;j} = t^i_{\tau|;j}\, y^a_{,i} + t^i_{\tau|} \sum_\nu \Omega_{\nu|ij}\, N^a_{\nu|} +$$
$$+ \sum_\nu c_{\nu\tau} [-\Omega_{\nu|jk}\, g^{kl}\, y^a_{,l} + \sum_\mu \vartheta_{\mu\nu|j}\, N^a_{\mu|}] +$$
$$+ \sum_\nu c_{\nu\tau|;j}\, N^a_{\nu|}, \tag{3.5}$$

where

$$\vartheta_{\mu\nu|i} = a_{\alpha\beta}\, N^a_{\nu|;i}\, N^\beta_{\mu|}. \tag{3.6}$$

(3.5) may be written as

$$\lambda^a_{\tau|;j} = (t^l_{\tau\ ;j} - \sum_\nu c_{\nu\tau}\, \Omega_{\nu|jk}\, g^{kl})\, y^a_{,l} +$$
$$+ \sum_\nu [t^i_{\tau|}\, \Omega_{\nu|ij} + c_{\nu\tau|;j} + \sum_\mu c_{\mu\tau}\, \vartheta_{\nu\mu|j}]\, N^a_{\nu|}, \tag{3.7}$$

or

$$\lambda^a_{\tau|;j} = \Sigma\, v_{\nu\tau|j}\, N^a_{\nu|} + q^l_{\tau|j}\, y^a_{,l}, \tag{3.8}$$

where

$$v_{\nu\tau|j} = t^i_{\tau|}\, \Omega_{\nu|ij} + C_{\nu\tau|;j} + \sum_\mu c_{\mu\tau}\, \vartheta_{\nu\mu|j}, \tag{3.9}$$

and

$$q^l_{\tau|j} = t^l_{\tau|;j} - \Sigma\, c_{\nu\tau|}\, \Omega_{\nu|jk}\, g^{kl}. \tag{3.10}$$

Generalized covariant derivative of (3.8) w. r. t. x^k yields

$$\lambda^a_{\tau|;jk} = \sum_\nu v_{\nu\tau|j;k}\, N^a_{\nu|} + \sum_\nu v_{\nu\tau|j}\, N^a_{\nu|;k} + q^l_{\tau|j;k}\, y^a_{,l} + q^l_{\tau|j}\, y^a_{;lk}. \tag{3.11}$$

Making use of (2.6) and (2.8), (3.11) reduces to

$$\lambda^a_{\tau|;jk} = \sum_\nu v_{\nu\tau|j;k}\, N^a_{\nu|} + \sum_\nu v_{\nu\tau|j}\, [-\Omega_{\nu|kh}\, g^{hp}\, y^a_{,p} +$$
$$+ \sum_\mu \vartheta_{\mu\nu|k}\, N^a_{\mu|}] + q^l_{\tau|j;k}\, y^a_{,l} + q^l_{\tau|j} \sum_\nu \Omega_{\nu|lk}\, N^a_{\nu|}. \tag{3.12}$$

If $R_{\alpha\beta\gamma\delta}$, $R^a{}_{\beta\gamma\delta}$ be the Riemann symbols of the first and second kind respectively for the tensor $a_{\alpha\beta}$ then

$$\lambda^a_{\tau|;jk} - \lambda^a_{\tau|;kj} = R^a_{\beta\gamma\delta}\, \lambda^\beta_{\tau|}\, y^\gamma_{,j}\, y^\delta_{,k}. \tag{3.13}$$

Making use of (3.12), (3.13) yields

$$R^{\alpha}_{\beta\gamma\delta}\,\lambda^{\beta}_{\tau|}\,y^{\gamma}_{,j}\,y^{\delta}_{,k} = \sum_{\nu} N^{\alpha}_{\nu|}\,[(v_{\nu\tau|j;k} - v_{\nu\tau|k;j}) +$$

$$+ \sum_{\mu}(\vartheta_{\nu\mu|k}\,v_{\mu\tau|j} - \vartheta_{\nu\mu|j}\,v_{\mu\tau|k}) + (q^{l}_{\tau|j}\,\Omega_{\nu|lk} - q^{l}_{\tau|k}\,\Omega_{\nu|lj})] +$$

$$+ y^{\alpha}_{,l}\,(q^{l}_{\tau|j;k} - q^{l}_{\tau|k;j}) + \Sigma\, v_{\nu\tau|j}\,\Omega_{\nu|kh}\,g^{hl}\,y^{\alpha}_{,l} - \Sigma\, v_{\nu\tau|k}\,\Omega_{\nu|jh}\,g^{hl}\,y^{\alpha}_{,l}. \quad (3.14)$$

Multiplying (3.14) throughout by $a_{\alpha\rho}\,N^{\rho}_{\sigma|}$ and summing on α, we have, by virtue of (2.3) and (2.4)

$$R_{\rho\beta\gamma\delta}\,N^{\rho}_{\sigma|}\,\lambda^{\beta}_{\tau|}\,y^{\gamma}_{,j}\,y^{\delta}_{,k} = (v_{\sigma\tau|j;k} - v_{\sigma\tau|k;j}) +$$

$$+ \sum_{\mu}(\vartheta_{\sigma\mu|k}\,v_{\mu\tau|j} - \vartheta_{\sigma\mu|j}\,v_{\mu\tau|k}) + q^{l}_{\tau|j}\,\Omega_{\sigma|lk} - q^{l}_{\tau|k}\,\Omega_{\sigma|lj}. \quad (3.15)$$

Similarly, if we multiply the equation (3.14) throughout by $a_{\alpha\rho}\,y^{\rho}_{,m}$ and sum on α, we have, on using (2.3) and (2.4),

$$\lambda^{\beta}_{\tau|}\,R_{\rho\beta\gamma\delta}\,y^{\gamma}_{,j}\,y^{\delta}_{,k}\,y^{\rho}_{,m} = g_{lm}\,[q^{l}_{\tau|j;k} - q^{l}_{\tau|k;j} + \Sigma\, v_{\nu\tau|k}\,\Omega_{\nu|jh}\,g^{hl} -$$

$$- \sum_{\nu} v_{\nu\tau|j}\,\Omega_{\nu|kh}\,g^{hl}], \quad (3.16)$$

or

$$R_{\rho\beta\gamma\delta}\,\lambda^{\beta}_{\tau|}\,y^{\gamma}_{,j}\,y^{\delta}_{,k}\,y^{\rho}_{,m} = (q_{\tau|mj;k} - q_{\tau|mk;j}) + \sum_{\nu}(v_{\nu\tau|k}\,\Omega_{\nu|mj} -$$

$$- v_{\nu\tau|j}\,\Omega_{\nu|mk}). \quad (3.17)$$

The equations (3.15) and (3.17) are the fundamental equations for the sub-space V_n of a Riemannian manifold V_m.

Multiplying the equations (3.8) by $a_{\alpha\rho}\,N^{\rho}_{\sigma|}$ and summing on ρ, we obtain

$$a_{\alpha\rho}\,N^{\rho}_{\sigma|}\,\lambda^{\alpha}_{\tau|;j} = v_{\sigma\tau|j}. \quad (3.18)$$

Equations (3.18) give expressions for the quantities $v_{\sigma\tau|j}$.

4. **Particular cases.** (*a*) Let us consider the congruences of curves which are such that the vector, with contravariant components $\lambda^{\alpha}_{\tau|}$ in the y's, of the unit vector in the direction of the curve of the congruence, is normal to V_m. Then

$$\lambda^{\alpha}_{\tau|} = N^{\alpha}_{\tau|},\ c_{\nu\tau} = \delta^{\nu}_{\tau}, \quad (4.1)$$

$$v_{\nu\tau|j} = \vartheta_{\nu\tau|j}, \tag{4.2}$$

$$q^l_{\tau|j} = -\Omega_{\tau|jh}\, g^{hl}. \tag{4.3}$$

Multiplying (4.3) throughout by g_{lm} and summing on l, we obtain

$$q_{\tau|mj} = -\Omega_{\tau|jm}. \tag{4.4}$$

Using (4.1), (4.2), (4.3), and (4.4) equations (3.15) and (3.17) yield

$$\begin{aligned} N^{\rho}_{\sigma|} N^{\beta}_{\tau|}\, y^{\gamma}_{,j}\, y^{\delta}_{,k}\, R_{\rho\beta\gamma\delta} = {} & (\vartheta_{\sigma\tau|j;k} - \vartheta_{\sigma\tau|k;j}) + \\ & + \sum_{\mu} (\vartheta_{\sigma\mu|k}\, \vartheta_{\mu\tau|j} - \vartheta_{\sigma\mu|j}\, \vartheta_{\mu\tau|k}) + \\ & + g^{hl}\, (\Omega_{\sigma|lj}\, \Omega_{\tau|kh} - \Omega_{\sigma|lk}\, \Omega_{\tau|jh}), \end{aligned} \tag{4.5}$$

and

$$\begin{aligned} N^{\beta}_{\tau|}\, y^{\gamma}_{,j}\, y^{\delta}_{,k}\, y^{\rho}_{,m}\, R_{\rho\beta\gamma\delta} = {} & (\Omega_{\tau|mk;j} - \Omega_{\tau|mj;k}) + \\ & + \sum_{\nu} (\vartheta_{\nu\tau|k}\, \Omega_{\nu|mj} - \vartheta_{\nu\tau|j}\, \Omega_{\nu|mk}). \end{aligned} \tag{4.6}$$

Equation (4.5) is the condition of integrability and equations (4.6) are the Codazzi equations of a V_n in a V_m (Eisenhart, 1949).

(*b*) If $m = n + 1$, the quantities $\vartheta_{\mu\nu|j}$ are zero, as follows from (3.6). Then (4.5) reduces to

$$N^{\rho}\, N^{\beta}\, y^{\gamma}_{,j}\, y^{\delta}_{,k}\, R_{\rho\beta\gamma\delta} = g^{hl}\, (\Omega_{lk}\, \Omega_{jh} - \Omega_{lj}\, \Omega_{kh}), \tag{4.7}$$

which is an identity since $R_{\alpha\beta\gamma\delta}$ is symmetric in index-pairs $\alpha\beta$ and $\gamma\delta$ (Eisenhart, 1949). (4.6) yileds, in this case,

$$N^{\beta}\, y^{\gamma}_{,j}\, y^{\delta}_{,k}\, y^{\rho}_{,m}\, R_{\rho\beta\gamma\delta} = \Omega_{mk;j} - \Omega_{mj;k}, \tag{4.8}$$

which are the Codazzi equations of a hypersurface V_n in V_{n+1} (Weatherburn, 1950).

(*c*) If the enveloping space is an Euclidean V_3 and the y's are Cartesian co-ordinates, (4.8) yields

$$\Omega_{mj,k} - \Omega_{mk,j} = 0,$$

which are the Codazzi equations for a surface (Eisenhart, 1947).

(*d*) If $m = n + 1$, (3.15) and (3.17) yield,

$$\lambda^{\beta}\, N^{\rho}\, y^{\gamma}_{,j}\, y^{\delta}_{,k}\, R_{\rho\beta\gamma\delta} = (v_{j;k} - v_{k;j}) + (q^l_j\, \Omega_{lk} - q^l_k\, \Omega_{lj}), \tag{4.9}$$

and

$$\lambda^{\beta} y^{\gamma}_{,j} y^{\delta}_{,k} y^{\rho}_{,m} R_{\rho\beta\gamma\delta} = (q_{mj;k} - q_{mk;j}) + (v_j \Omega_{mk} - v_k \Omega_{mj}), \quad (4.10)$$

where

$$v_j = t^i \Omega_{ij} + r_{;j}, \; c_{11} = r, \; q^l_j = t^l_{;j} - r \Omega_{jk} g^{kl}, \; t_i t^i = \sin^2 \phi, \; \theta_{11} = \phi. \quad (4.11)$$

Equations (4.9) and (4.10) are fundamental equations for the hypersurface of a Riemannian manifold (Mishra, 1952).

(*e*) If the enveloping space V_{n+1} is a Euclidean E_{n+1}, (4.9) and (4.10) yield

$$v_{j,k} - v_{k,j} = q^l_k \Omega_{lj} - q^l_j \Omega_{lk}, \quad (4.12)$$

and

$$q_{mj,k} - q_{mk,j} = v \; \Omega_{mk} - v_k \Omega_{mj}, \quad (4.13)$$

which are similar to the equations already obtained (Behari and Mishra, 1949) for a rectilinear congruence in a Euclidean E_3.

(*f*) If the enveloping space be a Euclidean E_3, (4.12) and (4.13) reduce to the equations for a rectilinear congruence through points on a surface (Behari and Mishra, 1949).

REFERENCES

1. R. Behari and R. S. Mishra : Some formulae in rectilinear congruences, *Proc. National Institute of Sci. India*, 15 (1949), 85-92.

2. L. P. Eisenhart : *An introduction to differential geometry with use of the tensor calculus*. (1947), 219.

3. L. P. Eisenhart : *Riemannian geometry* (1949), 162-63.

4. R. S. Mishra : Congruence of curves through points of a hyper-surface, *Ganita* 3 (1952), 37-40.

5. C. E. Weatherburn : *An introduction to Riemannian geometry and the tensor calculus*, (1950), 138, 162-63, 169-70.

ON THE TOTALLY GEODESIC SUB-SPACES IMBEDDED IN A RIEMANNIAN SPACE

By M. K. SINGAL and RAM BEHARI

1. **Introduction.** Ricci (1904) has established that *if a* V_{n+1} admits a hyper-surface with indeterminate lines of curvature for which $\Omega = 0$ (i.e. for a totally geodesic hyper-surface) the normal to V_n at a point is a Ricci principal direction for V_{n+1}, at points of V_n. The object of this paper is to obtain a generalization of this result for a sub-space V_n imbedded in a Riemannian V_m in the form: 'If a V_m admits a totally geodesic sub-space V_n the normals to V_n at a point are the Ricci principal directions for V_m.'

It is also proposed to show that if a V_{n+1} admits ∞^1 totally geodesic hyper-surfaces V_n, the tangential directions at any point are Ricci principal directions for V_{n+1}, if and only if, all the hyper-surfaces are Einstein spaces.

2. Let us consider a sub-space V_n of co-ordinates $x^i (i = 1, 2, \ldots, n)$ and metric $ds^2 = g_{ij}\, dx^i\, dx^j$ imbedded in a Riemannian $V_m (m > n)$ of co-ordinates $y^\alpha (\alpha = 1, 2, \ldots, m)$ and metric

$$ds^2 = a_{\alpha\beta}\, dy^\alpha\, dy^\beta. \tag{2.1}$$

We have then the relations

$$g_{ij} = a_{\alpha\beta}\, y^\alpha_{,i}\, y^\beta_{,j}, \tag{2.2}$$

and

$$a^{\alpha\beta} = g^{ij}\, y^\alpha_{,i}\, y^\beta_{,j}. \tag{2.3}$$

Let $N^\alpha_{\nu|}$ be the contravariant components in the y's of a system of $(m - n)$ unit normals to V_n; they satisfy the relations:

$$a_{\alpha\beta}\, N^\alpha_{\mu|}\, N^\beta_{\nu|} = \delta^\mu_\nu, \tag{2.4}$$

$$a_{\alpha\beta}\, y^\alpha_{,i}\, N^\beta_{\nu|} = 0. \tag{2.5}$$

Also,

$$y^{\alpha}_{;ij} = \sum_{\nu} \Omega_{\nu|ij} N^{\alpha}_{\nu|}, \tag{2.6}$$

where $\Omega_{\nu|ij}$ are the components of a symmetric tensor of the second order given by

$$\Omega_{\nu|ij} = a_{\alpha\beta}\, y^{\alpha}_{;ij}\, N^{\beta}_{\nu|}\,. \tag{2.7}$$

Also, by the Voss-Ricci equations of a V_n in a V_m (Eisenhart, 1949) we obtain,

$$\Omega_{\nu|ij,k} - \Omega_{\nu|ik,j} = \sum_{\tau} (\vartheta_{\tau\nu|k}\, \Omega_{\tau|ij} - \vartheta_{\tau\nu,j}\, \Omega_{\tau|ik}) + \bar{R}_{\alpha\beta\gamma\delta}\, y^{\alpha}_{,i}\, y^{\gamma}_{,j}\, y^{\delta}_{,k}\, N^{\beta}_{\nu|}, \tag{2.8}$$

where $\bar{R}_{\alpha\beta\gamma\delta}$ are the Riemann symbols of the second kind formed with respect to the metric of V_m, and

$$\vartheta_{\mu\nu|i} = a_{\alpha\beta}\, N^{\alpha}_{\nu|;i}\, N^{\beta}_{\mu|}.$$

3. **Totally geodesic sub-space V_n.** A sub-space V_n of an enveloping V_m is said to be totally geodesic if all the geodesics of V_n are geodesics of V_m. The necessary and sufficient conditions that a sub-space V_n may be totally geodesic are that

$$\Omega_{\nu|ij} = 0\,, \quad (\nu = n+1, \ldots, m). \tag{3.1}$$

4. For a totally geodesic sub-space the Voss-Ricci equations (2.8) yield, by virtue of (3.1),

$$\bar{R}_{\alpha\beta\gamma\delta}\, y^{\alpha}_{,i}\, y^{\gamma}_{,j}\, y^{\delta}_{,k}\, N^{\beta}_{\nu|} = 0. \tag{4.1}$$

Multiplying (4.1) throughout by g^{ik} and summing on i and k we have, after using (2.3),

$$\bar{R}_{\alpha\beta\gamma\delta}\, a^{\alpha\delta}\, y^{\gamma}_{,j}\, N^{\beta}_{\nu|} = 0\,, \text{ or } \bar{R}_{\beta\gamma}\, y^{\gamma}_{,j}\, N^{\beta}_{\nu|} = 0\,, \tag{4.2}$$

where $\bar{R}_{\beta\gamma}$ are the components of the Ricci tensor for V_m.

If we put

$$\bar{R}_{\beta\gamma}\, N^{\beta}_{\nu|}\, N^{\gamma}_{\nu|} = \rho\,, \tag{4.3}$$

it follows from (4.2) and (2.5) that

$$(\bar{R}_{\beta\gamma} - \rho a_{\beta\gamma})\, N^{\beta}_{\nu|}\, y^{\gamma}_{,j} = 0, \tag{4.4}$$

and

$$(\bar{R}_{\beta\gamma} - \rho a_{\beta\gamma})\, N^{\beta}_{\nu|}\, N^{\gamma}_{\nu|} = 0. \tag{4.5}$$

Since the m vector yields $y^{a}_{,i}$ and

$$N^{a}_{\nu|} \begin{pmatrix} i = 1, 2, 3, \ldots, n, \\ \nu = n+1, \ldots, m \end{pmatrix}$$

are independent, we have,

$$(\bar{R}_{\beta\gamma} - \rho a_{\beta\gamma})\, N^{\beta}_{\nu|} = 0. \tag{4.6}$$

From (4.6) it follows that *if a V_m admits a totally geodesic sub-space V_n, the normals to V_n at a point are Ricci principal directions w.r.t. V_m at points of V_n*, which is a generalization of the known result that if a V_{n+1} admits a hyper-surface V_n with indeterminate lines of curvature and $\Omega = 0$, the normal to V_n at a point is a Ricci principal direction for V_{n+1}, at points of V_n. (*Vide* Eisenhart, *Riem. Geo.* p. 181).

5. For a Riemannian V_n embedded in a V_{n+1}, we know (Eisenhart, 1949) that,

$$R_{ijkl} = (\Omega_{ik}\, \Omega_{jl} - \Omega_{il}\, \Omega_{jk}) + \bar{R}_{\alpha\beta\gamma\delta}\, y^{\alpha}_{,i}\, y^{\beta}_{,j}\, y^{\gamma}_{,k}\, y^{\delta}_{,l}, \tag{5.1}$$

where the symbols have the usual meaning.

If V_n be totally geodesic, $\Omega_{ij} = 0$, $(i, j, = 1, 2, \ldots, n,)$, (3.1) then gives

$$R_{ijkl} = \bar{R}_{\alpha\beta\gamma\delta}\, y^{\alpha}_{,i}\, y^{\beta}_{,j}\, y^{\gamma}_{,k}\, y^{\delta}_{,l}. \tag{5.2}$$

Multiplying (5.2) throughout by g^{il} and summing on i and l, we obtain, by virtue of $g^{il}\, R_{ijkl} = R_{jk}$ and (2.5),

$$R_{jk} = \bar{R}_{\beta\gamma}\, y^{\beta}_{,j}\, y^{\gamma}_{,k}. \tag{5.3}$$

Multiplying (5.3) throughout by $y^{\delta}_{,l}\, g^{kl}$ and summing on k we obtain

$$R_{jk}\, y^{\delta}_{,l}\, g^{kl} = \bar{R}_{\beta\gamma}\, y^{\beta}_{,j}\, y^{\gamma}_{,k}\, y^{\delta}_{,l}\, g^{kl},$$

or

$$R_{jk}\, y^{\delta}_{,l}\, g^{kl} = a^{\gamma\delta}\, \overline{R}_{\beta\gamma}\, y^{\beta}_{,j}, \tag{5.4}$$

by virtue of (2.5). Multiplying (5.4) throughout by $a_{\delta\lambda}$ and summing on δ, we obtain

$$\begin{aligned} R_{jk}\, y^{\delta}_{,l}\, g^{kl}\, a_{\delta\lambda} &= a^{\gamma\delta}\, a_{\delta\lambda}\, \overline{R}_{\beta\gamma}\, y^{\beta}_{,j} \\ &= \delta^{\gamma}_{\lambda}\, \overline{R}_{\beta\gamma}\, y^{\beta}_{,j} \\ &= \overline{R}_{\beta\lambda}\, y^{\beta}_{,j}. \end{aligned} \tag{5.5}$$

If V_n be an Einstein space,

$$R_{jk} = \rho g_{jk}, \tag{5.6}$$

whence (5.5) becomes

$$\rho g_{jk}\, y^{\delta}_{,l}\, g^{kl}\, a_{\delta\lambda} = \overline{R}_{\beta\lambda}\, y^{\beta}_{,j},$$

or

$$\rho a_{\delta\lambda}\, y^{\delta}_{,l}\, \delta^{l}_{j} = \overline{R}_{\beta\lambda}\, y^{\beta}_{,j},$$

or

$$\rho a_{\beta\lambda}\, y^{\beta}_{,j} = \overline{R}_{\beta\lambda}\, y^{\beta}_{,j},$$

or

$$(\overline{R}_{\beta\lambda} - \rho a_{\beta\lambda})\ \ y^{\beta}_{,j} = 0, \tag{5.7}$$

showing that *the tangential directions $y^{\beta}_{,j}$ are Ricci principal directions for the enveloping V_{n+1}.*

Conversely, if the tangential directions $y^{\beta}_{,j}$ are Ricci principal directions for the enveloping V_{n+1}, we have

$$(\overline{R}_{\beta\lambda} - \rho a_{\beta\lambda})\, y^{\beta}_{,j} = 0, \tag{5.8}$$

where

$$\rho = \overline{R}_{\beta\lambda}\, y^{\beta}_{,j}\, y^{\lambda}_{,j}; \tag{5.9}$$

from (5.5) and (5.8) we have

$$R_{jk}\, y^{\delta}_{,l}\, g^{kl}\, a_{\delta\lambda} = \rho a_{\beta\lambda}\, y^{\beta}_{,j}. \tag{5.10}$$

Multiplying (5.10) throughout by $y^{\lambda}_{,p}$ and summing on λ we have, by virtue of (2.4),

$$R_{jk}\, y^{\delta}_{,l}\, g^{kl}\, a_{\delta\lambda}\, y^{\lambda}_{,p} = \rho a_{\beta\lambda}\, y^{\beta}_{,j}\, y^{\lambda}_{,p},$$

or

$$R_{jk}\,\delta^k_p = \rho g_{jp}\,,$$

or

$$R_{jp} = \rho g_{jp}\,,$$

showing that the *hyper-surface must be an Einstein space. Hence, if a* V_{n+1} *admits* ∞^1 *totally geodesic hyper-surfaces* V_n, *the tangential directions at any point are Ricci principal directions if and only if all the hyper-surfaces are Einstein spaces.*

REFERENCES

1. L. P. Eisenhart: *Riemannian geometry* (1949), 184-85.

2. Ricci: Direzioni e invarianti principali in una varieta qualanque, *Atti del. Reale Inst. Vaneto*, 63 (1904), 1233-39.

3. C. E. Weatherburn: *Riemannian geometry and the tensor calculus* (1950), 162-70.

University of Delhi

SYMPOSIUM ON ASTROPHYSICS

Chairman: PROFESSOR D. S. KOTHARI

Dr. P. L. Bhatnagar gave a brief account of the work done on magneto-hydrodynamics by Alfven, Astrom, Lundquist and others, pointing out the growing importance of the subject in the context of astro-physics. He discussed how one could take into account in a natural way the effects of compressibility, finite electric conductivity and transfer of energy by working through the molecular aspect of an ionized gas.

Dr. Vainu Bappu summarized the data on the chromospheric phenomena in Wolf-Rayet stars and the stars of spectral types F, G, K, M. He discussed the nature of H and K emission lines of *Ca II* observed in many F, G, K, M stars and suggested that their origin may possibly be explained in terms of large scale prominence activity or calcium flocculi. He then pointed out that there are certain basic differences between the appearances of the K_3-profiles in the integrated spectrum of the sun and that of the other stars and suggested that these can be explained in terms of the high temperature corona of the sun. He reported that the K_3-profiles of the stars analysed by him indicate a large scale turbulent activity particularly in supergiant chromospheres, the trend increasing from luminosity class V to luminosity class I. He then passed on to the Wolf-Rayet stars and explained that recent investigations suggest a need for considerable revision of the Beal's picture of the atmospheres of W-stars.

Dr. R. S. Kushwaha spoke on the pulsations of stars with variable ratio of specific heats, and observed that one of the crucial points in the pulsation theory is the proper interpretation of the relation : *Period of pulsation* $\times$ (*mean density*)$^{\frac{1}{2}} = K$, a constant. The existing theories give a low value for K. He pointed out that this discrepancy can be removed in the case of certain stars by taking a suitable law of variation of the ratio of specific heats.

Dr. M. A. Naqvi spoke on the physical conditions in solar corona and pointed out that although many emission lines have been reported to be visible in the coronal spectrum, only about twenty-four of them are definitely known to arise in the inner corona. Edlén has made an attempt to identify these lines. According to him a large number of them is due to the highly ionized atoms of iron, nickel, calcium and argon, but some of the identifications are doubtful and some of them still remain unidentified. He, then, reported that Kundu, Garstag and he are carrying on this work, though some of their conclusions are bound to be uncertain as no laboratory data are available about the spectra of the highly ionized atoms.

The following speakers could not present their papers on account of the shortage of time :

(1) Dr. F. C. Auluck: Fragmentation and size distribution of stars,

(2) Mr. Pyare Lal : Pressure ionization and the constitution of planets.

Dr. Gagan Behari Bandopadhyay had sent the synopsis of his talk on " Homologous Radial Motion of a gas sphere and an interesting equation related to it ", but was unable to attend the Conference.

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ABSTRACTS

ALGEBRA AND THEORY OF NUMBERS

D. K. Sen : *A note on Newton's method for calculating roots of* $f(x) = 0$

Regarding Newton's formula $x_{r+1} = x_r - f(x_r)/f'(x_r)$ as derived by a method of geometrical iteration, one sees that the success of Newton's method depends upon the convergence of a sequence of points and not on the smallness of the difference between the initial and the exact value of the root. There are several advantages of the geometrical approach. In the first place, it leads to the conclusion that Newton's method can be applied in all cases except, at times, when $f'(x)$ is infinite at, or very close to, the root. Secondly, one obtains the following useful rules to be used in conjunction with Newton's formula; these rules are valid when neither $f'(x)$ nor $f''(x)$ is zero or infinite at the root. I. If $f'(x)$ and $f''(x)$ preserve their signs in a (small) neighbourhood containing the root c, then x_{r+1} is less than c, when $f'(x)$ and $f''(x)$ have opposite signs, and x_{r+1} is greater than c, when $f'(x)$ and $f''(x)$ have the same sign, irrespective of whether x_r is less or greater than c, II. When $f'(x)$ and $f''(x)$ have opposite signs and x_r is less than c, or when $f'(x)$ and $f''(x)$ have the same sign and x_r *is greater than* c, then x_{r+1} lies on the same side of c as x_r and is nearer c than x_r. These rules are of great help in shortening the calculation, in determining whether the value obtained at any stage is less or greater than the root and in being able to use the formula properly even in difficult cases. These are illustrated with the help of an example.

M. V. Jambunathan : *Inverses of successively bordered matrices*

Let A_m stand for the matrix $\| a_{ij} \|$, $(i = 1, 2, \ldots m)$, and let A_m be non-singular for all values of m considered. The matrix A_{m+1} is obtained by 'bordering' the matrix A_m with one additional column and one additional row. In several situations, the need arises for calculating the inverses of A_{m-1} and A_{m+1} after one has calculated and obtained the inverse of A_m. The procedure for obtaining these inverses is somewhat involved but certain methods have been devised to compute them without the necessity of working over again starting from the original matrix. For instance, when the inverse of A_3 has been calculated as $\| c_{ij} \|$, $(i, j = 1, 2, 3)$, the inverses of A_2 can be expressed as $\| c'_{ij} \|$, $(i, j = 1, 2)$ where $c'_{ij} = c_{ij} - c_{i3}\, c_{3j}/c_{33}$. The method of 'sweep-out' which is very effective but perhaps not widely employed in calculating inverses of matrices can be extended to partitioned matrices and the above results deduced therefrom. This paper presents a schematic method of applying the 'sweep-out' which has the unique advantage of enabling one to read off, at any stage of the work, the inverses of all the matrices of lower order, while at the same time no step is superflous when the inverse of the next higher order is to be obtained in the same process.

M. V. Jambunathan: *Calculation of the covariance matrix in some special cases*

In statistical estimation we meet with the following problem : If $y_1, y_2, \ldots y_n$ be n stochastic variates with common variance σ^2, and the observational equations

are given by $E(\eta) = \tau A$, where $\eta = (y_1, y_2, \ldots y_n)$, $\tau = (t_1, t_2, \ldots t_k)$, $A = \| a_{ij} \|$, $i = 1, 2, \ldots k$, $j = 1, 2, \ldots n$, the a's being known constants, it is required to estimate the k unknown parameters $t_1, t_2, \ldots t_k$. The solution is simple when the variates are uncorrelated. If (AA') be non-singular, then the unknown parameters are given explicitly by the relation $\tau = \eta A' (AA')^{-1} = (AA')^{-1} A\eta'$. But when the stochastic variates are correlated with dispersion matirx Λ, then by Bose's fundamental theorem of linear estimation for correlated variates, the best unbiased estimate of an estimable parametric function $l\tau'$ is obtained as $l(A\Lambda^{-1}A')^{-1}A\Lambda^{-1}\eta'$, provided that $(A\Lambda^{-1}A')$ is non-singular. The particular case in which $V(y_i) = \sigma^2$ for all i, and cov $(y_i, y_j) = \rho\,\sigma^2$ for all $i \neq j$, is of importance. In this case it is shown that the inverse of $(A\Lambda^{-1}A')$ is equal to $A_n \Lambda A'_n$, where A_n stands for $1/n.A$. This method can also be applied to the case of estimation of parameters for a stratified sample.

M. V. Jambunathan : *Use of continued fractions in the evaluation of determinants*

The method of evaluating determinants by pivotal condensation is well known. The present paper shows that the application of this method to a determinant whose leading diagonal has each element equal to a, the truncated parallel diagonals immediately above and below the leading diagonal having each element equal to unity and all the remaining elements being zero, leads to a triangular determinant in which the leading diagonal elements are the successive convergents of a continued fraction. The method of evaluating the determinant in terms of the successive convergents of the related continued fraction is illustrated by a suitable example.

D. R. Kaprekar : *On self numbers*

If d_1 represents the sum of the digits of a positive integer N_1, then $N_1 + d_1 = N_2$ will be the generated number of N_1. Similarly, if d_2 is the sum of the digits of N_2 we get a number $N_3 = N_2 + d_2$. The series N_1, N_2, N_3 is called a digitadition series, e.g. 31, 35, 43, 50, 55 .... Conversely given a number N is there an M such that N equals M plus the sum of its digits ? If there is no such M, N is called self number. For example 31 is self number. Many primes like 31, 53, 97, 233, 727, 929 are self numbers. Some demlo numbers like 132, 143, 154...and some big numbers like 1952, 874531 are also self numbers. If S and T are two numbers, their digitadition series may have one point common. For example the digitadition series of 1 and 5 meet at the common point 620. A chart of the digitadition series of 1 is constructed till the term 1520 and in this range the digitadition series of nearly 70 different self numbers come to meet to form common points. The digitadition series of 121 meets in the range at 1003 with 15 digitadition series of different self numbers and nearly 5 subjunctions. There are several types of series shown here. Formulae are derived to find the sum of all the generated numbers in any range neglecting self numbers.

K. Subbarao : *Some properties of Fibonacci Numbers* III

Following the notation of my recent papers bearing the same title, I consider in this paper the following Fibonacci series, 1, 3, 4, 7, 11, 18 ... , which is also known as the Lucas series and whose nth term is given by

$$v_n = \left\{ (1 + \sqrt{5})/2 \right\}^n + \left\{ (1 - \sqrt{5})/2 \right\}^n .$$

I prove THEOREM 1. *For* $k > 1$ *and* $m > m_0$, (1) $v_{2km} < v_{2m}{}^k < v_{2km+1}$, (2) $v_{2km+k-1} < v_{2m+1}{}^k < v_{2k+m+k}$.

THEOREM 2. *Between* $v_{2m}{}^k$ *and* $v_{2m+1}{}^k$, *there lie exactly* $(k-1)$ *Lucas numbers and* $(k+1)$ *numbers lie between* $v_{2m+1}{}^k$ *and* $v_{2m+2}{}^k$.

THEOREM 3. *There are infinitely many Lucas numbers each of which is* $\equiv 1$ (*mod.* p), *where* p *is a given prime.*

The following problems are also considered :

(1) to find the sum of the kth powers of the first n Fibonacci numbers : (2) to find the sum of the kth powers of the first n Lucas numbers ; (3) to find the last digit of a Lucas number ; (4) to give an alternative proof of Euclid's Theorem that the number of primes is infinite.

M. S. Srinivasan : *Correspondence between integers and rational numbers*

The positive rational numbers can be shown to be enumerable by set theoretic methods. But these are of no use for arithmetic as the order of the rational numbers does not enter into the enumeration.

The method of enumeration considered here is by means of median series M_i, where

$$M_0 \ldots \quad \frac{0}{1}, \frac{1}{0} \qquad\qquad M_1 \ldots \quad \frac{0}{1}, \frac{1}{1}, \frac{1}{0}$$

$$M_2 \ldots \quad \frac{0}{1}, \frac{1}{2}, \frac{1}{1}, \frac{2}{1}, \frac{1}{0}$$

$$M_3 \ldots \quad \frac{0}{1}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{1}{1}, \frac{3}{2}, \frac{2}{1}, \frac{3}{1}, \frac{1}{0} \text{ etc.}$$

It can be shown that the fractions of M_i are irreducible and are in natural order and $M_0 \subset M_1 \subset M_2 \subset M_3 \subset \ldots.$

Any positive rational number ρ occurs for the first time as the $2k$th term in a certain M_r and does not arise again through mediation. We may call (ν, k) the *median co-ordinates* of the rational number ρ. The two problems which arise, viz " given k, ν to find the rational number ρ and conversely " can be answered by means of the two arithmetic *functions*, $\tau(N)$ of Hermes and $\xi(p, q)$, the sum of the quotients in the g.c.d. algorithm for p, q. The same problems for the rational numbers between two given rational numbers ν, ν' can be answered, by using a transformation.

ANALYSIS

Mansoor Ahmad : *On integral functions of integral order*

S. M. Shah [(1). *Jour. Ind. Math. Soc.* 5 (1941), 179-188 ; (2) *Jour. London Math. Soc.* 15 (1940), 25-31] has proved two theorems on integral functions of integral order, when the genus of the function is the same as that of its canonical product. We prove a result of the same type for integral functions of greater order.

I. If $X(z) = Y(z).\ f(z)$ be an integral function, where $f(z)$ is a canonical product of order $p+1$ and genus p, and $Y(z)$ is any integral function of order greater than $p+1$, and if

$$\lambda(r) = \left\{ r^{-p}\mu(r) \right\}^{a},$$

where $a > 1$, and $\log M(r, Y) = O\left\{ \mu(r) \right\}$, $\mu(r)$ being a steadily increasing function of r, then

$$\liminf_{r\to\infty} \frac{\lambda^{p}(r).\log M(r, X)}{r^{p}.\, n\left\{\lambda(r)\right\}.\ \phi\left\{\lambda(r)\right\}} = 0, \tag{A}$$

provided that a constant $a\ (> 1)$ exists such that

$$\liminf \frac{n(r)}{r^{p+1/a}} > 0.$$

II. If $X(z) = Y(z).\ f(z)$ is an integral function, where $f(z)$ is a canonical product of order and genus $p \geqslant 1$, and $Y(z)$ is any integral function of order greater than p, and if

$$\lambda(r) = \exp\left\{ r^{-p}.\ \mu(r) \right\},$$

then (A) holds, provided that

$$\liminf \frac{n(r)}{r^{p}} > 0.$$

C. N. Srinivasiengar and B. N. Mukherjee: *On a generalization of Bateman's K-function*

Some generalizations of Bateman's K-function have been given by other writers. In this paper, we introduce the function

$$\overline{K}_{2n}^{(m)}(x) = \frac{(-1)^{n}\, x^{m+1} e^{bx}}{n\,!}\ \frac{d^{n}}{dx^{n}}\, (e^{-ax}\, x^{n-m-1}),$$

$$\overline{K}_{0}^{(m)}(x) = e^{-(a-b)x},$$

where $x > 0$, $a > b > 0$, m is zero or a positive integer $< n-1$. This reduces to Bateman's function $K_{2n}(x)$ when $a = 2$, $b = 1$, $m = 0$, and has interesting properties analogous to those of $K_{2n}(x)$. It can also be connected with other well-known functions. A few properties are listed below :

$$\overline{K}_{2n}^{(m)}(x) + \overline{K}_{2n-2}^{(m)}(x) = \overline{K}_{2n}^{(m+1)}(x), \tag{1}$$

$$\left.\begin{aligned} \overline{K}_{2n}^{(m)}(x) &\doteqdot \frac{a^{m+1}p}{(p+a-b)^{m+2}} \left(\frac{b-p}{p+a-b} \right)^{n-m-1}, \quad n \geqslant 1 \\ \overline{K}_{0}^{(m)}(x) &\doteqdot \frac{p}{p+a-b} \end{aligned}\right\} \tag{2}$$

$$\overline{K}_{2n}^{(m)}(x) = (-1)^{n-m-1}\, \frac{(n-m-1)\,!}{n\,!}\, e^{-(a-b)x} a^{m+1} x^{m+1} L_{n-m-1}^{(m+1)}(ax), \tag{3}$$

$$e^{(a-b)x} \sum_{n=0}^{\infty} \frac{(-1)^{n} t^{n} \overline{K}_{2n+2m+2}^{(m)}(x)}{n\,!} = (a/t)^{m+1} e^{t} x^{(m+1)/2} J_{m+1}(2\sqrt{(atx)}), \tag{4}$$

$$e^{(a-b)x}\overline{K}_{2n}^{m}(2x) = \sum_{r=0}^{n-m-1} 2^{m+1+r} \binom{n-m-1}{r} \overline{K}_{2m+2r+2}^{(m)}(x), \tag{5}$$

$$\overline{K}_{2n}^{m}(x+y) = \sum_{r=0}^{n} \overline{K}_{2r}(y)\ \overline{K}_{2n-2r}^{(m)}(x). \tag{6}$$

Defining $\overline{K}_{2n}^{(0)}(x)$ as $\overline{K}_{2n}(x)$, we have

$$\sin cx = e^{(c-b)} \sum_{r=0}^{\infty} (-1)^r \overline{K}_{4r+2}(x), \tag{7}$$

$$\cos cx = e^{(c-b)} \sum_{r=0}^{\infty} (-1)^r \overline{K}_{4r}(x). \tag{8}$$

Giving particular values to the constants, many of these reduce to known results about the K-function. The following results of the K-function are probably new.

$$e^{2x}K_{2n}(2x) = 2\ K_2(x) + 2^2(n-1)\ K_4(x) + \frac{2^3(n-1)\ (n-2)}{2\,!}\ K_6(x) + \ldots + 2^n K_{2n}(x), \tag{9}$$

$$\frac{x^r}{r\,!} = \sum_{n=0}^{\infty} nK_{2n}(x) \sum_{l=0}^{r} \frac{\Gamma\,(n+r-l)}{l\,!\,(n-l)\,!\,(r-l)\,!}\,. \tag{10}$$

Sulaiman Kerawala : *The integration of the Darboux-Riccati equation for the general helix*

The integration of the Serret-Frenet equations can be reduced to the integration of the Darboux-Riccati equation, but few cases are known in which its integration can be given explicitly. Recently, Pöschl has shown that this can be done for a helix on the right circular cylinder or on the right circular cone. The purpose of the present note is to give the integration for the general helix.

S. M. Shah and S. K. Singh : *On Nevanlinna exceptional values of meromorphic functions*

This paper is in continuation of three papers : [S. M. Shah : Compositio Mathematica, 9, (1951), 227-38. S. M. Shah: Duke Math. Journal, 19 (1952), 585-93. S. M. Shah & S. K. Singh : Mathematische Zeit., 59 (1953), 88-93]. We prove here some results for the class of meromorphic functions for which the total Nevanlinna defect is attained. The results proved are :

I. If a meromorphic function $F(z)$ has only finite N exceptional values $(a_\nu)_1^p$, with

$$\sum_1^p \delta(a_\nu) = 2,\ (p > 1),\ \text{then}$$

$$T(r, F') \sim 2\ T(r, F).$$

II. If a meromorphic function $F(z)$ has N exceptional values ∞ and $(a_\nu)_1^p$ with $\delta(\infty) = 1$ and $\sum_1^p (a\delta_\nu) = 1$, then

$$T(r, F') \sim T(r, F).$$

Analogous results on relative defect values, as defined by H. Milloux, can also be proved.

S. K. Singh : *On the magnitude of the maximum term of the derivative of an entire function*

Let $f(z)$ be an entire function and let $\mu(r)$ be the maximum term and $\nu(r)$ its rank for $|z| = r$. Let $\mu^{(k)}(r)$ and $\nu^{(k)}(r)$ correspond to $f^{(k)}(z)$.

I prove in this paper :

I. If $f(z)$ be an entire function of order $\rho < 1$, then

$$\frac{r^p \mu^{(k)}(r)}{\mu(r)} \to 0 \text{ as } r \to \infty \quad (k = 1, 2, \ldots),$$

where $p > k(1-\rho)$.

II. If $f(z)$ be an entire function of lower order $\lambda > 1$, then

$$\frac{\mu^{(k)}(r)}{\mu(r) r^p} \to \infty \text{ as } r \to \infty \quad (k = 1, 2, \ldots),$$

where $p < k(\lambda - 1)$.

From the above I deduce the following results of S. M. Shah [J. Univ. of Bombay, XIII, pt. III, 1944 ; Bull. Amer. Math. Soc., 53, 1947, pp. 1156-63] :

$$\lim \begin{matrix} \sup \\ \inf \end{matrix} \frac{\log \left\{ r\, M'(r)/M(r) \right\}}{\log r} = \begin{matrix} \rho \ (0 \leqslant \rho \leqslant \infty) \\ \lambda \ (0 \leqslant \lambda \leqslant \infty), \end{matrix}$$

and I also deduce Valiron's inequality $M'(r) < M(r)\, r^{\rho - 1 + \epsilon}$.

A. M. Chak : *Some theorems in operational calculus*

Recently some authors have tried to investigate the properties of $f(t)$ which are reflected in $\phi(p)$ when $\phi(p)$ is the Laplace transform of $f(t)$ in the sense of Carson. In a paper in the Acta Math. (Vol 88, 1952) Mitra and Bose have proved that if $t^{-n-1/2} f(t)$ is self-reciprocal in the Hankel transform of order n, then $t^{n-1/2}\, \phi(t)$ is also self-reciprocal in the Hankel transform of order n Re $(n) \geqslant -1/2$. The object of this paper is to generalize this result and other results of a similar nature. The main theorem runs as follows : If $f(t).t^{-(m+1)/2}$ is T_g, then $\phi(t). t^{(m-1)/2}$ is T_g, provided $g(t)$ is H_{m+1} and conversely. Here we use the following definition : A function $f(t)$ is said to be T_g if it satisfies the relation

$$f(t) = \int_0^\infty g(xt)\, f(x)\, dx.$$

If, in particular,

$$g(t) = \int_0^\infty J_{m+1}\, (2(xt)^{1/2})\, g(x)\, dx,$$

we say that $g(t)$ is H_{m+1}.

A. Sharma and H. M. Srivastava : *Certain functional relations*

Erdelyi has shown a method of finding recurrence relations and expansions of functions which are defined as Hankel transforms of any given function. The object of this paper is to consider a function $f_{\mu,\nu}{}^{\lambda}(t)$ defined by

$$f_{\mu,\nu}{}^{\lambda}(t) = \int_a^b (st)^{\nu/2} J_\nu{}^{\lambda}(st^{\lambda})\, s^{\mu}\, F(s)\, ds,$$

where $F(s)$ is an arbitrary function and λ, μ, ν belong to the range of values for which the integral converges, J_ν^λ being the Bessel-Maitland function. On taking $F(s) = \exp\,[-s^{1/\lambda}]$ we define a function $M_{\mu+1/2,\nu/2}^{\lambda}(t)$ which has many properties analogous to Whittaker's $M_{k,m}$ function.

In particular for $\mu = \frac{\nu+m}{\lambda} - \frac{\nu}{2} - 1 + \frac{1}{\lambda}$, we get for $F(s) = \exp\,[-s^{1/\lambda}]$,

$$f_{\mu,\nu}^{\lambda}(t) = \exp\,[-t^\lambda].\quad L_{m,\lambda}^{\nu}(t^\lambda),$$

where $L_{m,\lambda}^{\nu}(t)$ is a polynomial of degree m, with the associated generating function

$$\frac{1}{(1-u)^{\nu+1}}\,.\,\exp\left[\omega - \frac{\omega}{(1-u)^\lambda}\right] = \sum_{m=0}^{\infty} u^m L_{m,\lambda}^{\nu}(\omega).$$

For $\lambda = 1$, we get the generalised Laguerre Polynomials.

H. M. Srivastava: *Some chains of Laplace transforms*

Recently a number of papers have come out wherein rules are given for deriving operational images in two varieties. The object of this paper is to give some chains of Laplace transforms in one variable which imply an operational image in two-variables :

Rule (i). If $f(p) \subset h(x)$, $\frac{1}{p^{\lambda-1}}\, h\left(\frac{1}{p}\right) \subset g(x)$,

$$(1/p^{2m+\lambda-1})\, g(p) \subset \phi(x) \text{ and } x^{2m+\lambda-1}\,\phi(1/x) \supset H(p),$$

then $$(xy)^m\,(x/y)^{\lambda-1+m}\,H(x/y) \supset\supset (1/p^{2m})\,f(q/p).$$

Rule (ii). If $f(p) \subset h(x)$, $(1/p^{\lambda-1})\,h(1/p^\mu)$, $\subset g(x)$,

$$(1/p^{m+(\lambda-1)/\mu})g(1/p^\mu) \subset \phi(x) \text{ and } x^{\lambda+m\mu-1}\,\phi(1/x^\mu) \supset H(p),$$

then $$(1/p^m)\,f(q/p) \subset\subset (-1)\,(xy)^{m/2+(\lambda-1)/\mu}\,H(x^{1/\mu}/y^{1/\mu})$$

Rule (iii). If $H(p) \subset x^{\nu-1} f(x)$, $\phi(p) \subset x^{\lambda+\mu-1} f(x^\mu)$,

then $$x^{\lambda/\mu}\,y^{\nu-1}\,h(x^{1/\mu}y) \supset\supset (-1)\,p^{1-\lambda+\nu/\mu}\,H(p^{1/\mu}q).$$

A number of interesting examples are given. Several known images listed in Doetsch and Völcker's 'Zwei dimensionale Laplace Transform......' follow easily from these rules, besides a number of new operational images in two variables.

Y. V. Thosar : *Associated function analogues of a Legendre function formula*

In this paper we obtain two analogues of a well-known Legendre function formula [Hobson : *Spherical and Ellipsoidal Harmonics* 1931, p. 51, (51)]. The first analogue is obtained by successive differentiation and the second by investigation of Legendre's associated equation. We then get a differential equation for a polynomial of degree $(n-1)$ that arises in the course of the above investigation. Finally, we derive a finite summation formula for that polynomial.

P. K. Menon: *A generalization of Jocobi's fundamental formulae in ϑ-fuctions*

The function $\vartheta(x, a, \rho \mid \tau)$ is defined by

$$\vartheta(x, a, \rho \mid \tau) = \sum_{n=-\infty}^{\infty} \rho^n q^{(n+a)^2} e^{2\pi i (n+a)x},$$

where $q = e^{i\pi\tau}$, imaginary part of τ being greater than zero, and ρ is an mth root of unity so that in the usual notation of ϑ-functions we have

$$\begin{aligned}\vartheta_3(x \mid \tau) &= \vartheta(x, 0, \quad 1 \mid \tau)\\ \vartheta_4(x \mid \tau) &= \vartheta(x, 0, -1 \mid \tau)\\ \vartheta_2(x \mid \tau) &= \vartheta(x, \tfrac{1}{2}, \quad 1 \mid \tau)\\ \vartheta_1(x \mid \tau) &= i\, \vartheta(x, \tfrac{1}{2}, -1 \mid \tau).\end{aligned}$$

A group theoretic method is developed by which we prove the following

THEOREM. Let a_k, b_k, c_k $(k = 1, 2, \ldots, n)$ be $3n$ integers such that $a_k, b_k, > 0$ $(k = 1, 2, \ldots, n)$, $(a_1, a_2, \ldots, a_n) = 1$, $(b_1, b_2, \ldots, b_n) = 1$, and

$$\sum_{k=1}^{n} b_k c_k = dc,$$

c being an integer and where d is given by

$$\sum_{k=1}^{n} a_k b_k = l = \begin{cases} d \text{ if } l \text{ is odd,} \\ 2d \text{ if } l \text{ is even;}\end{cases}$$

and let ρ be a dth root of unity. Also let z_k, z_k' $(k = 1, 2, \ldots, n)$ be complex variables connected by the relations

$$\frac{z_1 + z_1'}{2a_1} = \frac{z_2 + z_2'}{2a_2} = \ldots = \frac{z_n + z_n'}{2a_n} = \frac{\sum_{k=1}^{n} b_k z_k}{l},$$

l being the number defined above. Then, for l even,

$$\sum_{p=0}^{d-1} \rho^{p+c} \prod_{k=1}^{n} \vartheta \left\{ \frac{b_k}{a_k} z_k, \frac{a_k p + c_k}{d}, \rho^{bk} \middle| \frac{b_k}{a_k} \tau \right\}$$
$$= \sum_{p=0}^{d-1} \rho^{p} \prod_{k=1}^{n} \vartheta \left\{ \frac{b_k}{a_k} z_k', \frac{a_k p - c_k}{d}, 1 \middle| \frac{b_k}{a_k} \tau \right\};$$

and, for l odd,

$$\sum_{p=0}^{d-1} \rho^{2c+p} \prod_{k=1}^{n} \vartheta \left\{ \frac{b_k}{a_k} z_k, \frac{p a_k + c_k}{d}, \rho^{2bk} \middle| \frac{b_k}{a_k} \tau \right\}$$
$$= \sum_{p=0}^{d-1} \rho^{p} \prod_{k=1}^{n} \vartheta \left\{ \frac{b_k}{a_k} z_k', \frac{p a_k - c_k}{d}, 1 \middle| \frac{b_k}{a_k} \tau \right\}.$$

Jocobi's formulae are obtained by taking

$$n = 4,\ a_k = b_k = 1,\ (k = 1, 2, 3, 4),$$

so that $l = 4 = 2d$ and $\rho = \pm 1$, and where c_1, c_2, c_3, c_4 take the values 0, 1, subject to the condition that an even number of them are zeros.

T. P. Srinivasan: *On measurable sets*

A *measure space* for Segal is a pair (R, r) where R is a ring of subsets of a set X and r, a *finite* measure on R such that, if $A_n \in R$, $n = 1, 2 \ldots$, and $\sum_{n=1}^{\infty} r(A_n) < \infty$, then $\bigcup_{n=1}^{\infty} A_n \in R$. We define with Segal that a set E is *measurable* if $E \cap A \in R$ for every set $A \in R$. If, for a measurable set E, we define $\bar{r}(E) = \sup \left\{ r(A)/A \in R, A \subset E \right\}$, then it is shown that r is a measure on the class $\bar{M}$ of measurable sets and is an extension of r on R. If now r^* is the outer measure induced by r and M^* the class of r^* - measurable sets, in the sense of Caratheodory, the precise relationship of the class $\bar{M}$ to M^* is not found in the literature.

We show here that if $\bar{\bar{r}}$ is the " completion " of $\bar{r}$ and $\bar{\bar{M}}$ the underlying ring for $\bar{\bar{r}}$ then $\bar{\bar{M}} = M^*$. We further show that if we define the set function r_* on any set E by : $r_*(E) = \sup \left\{ r(A)/A \subset E, A \in R \right\}$ and define E to be r_* - measurable if for every set A for which $r_*(A) < \infty$ the relation $r_*(A) = r_*(A \cap E) + r_*(A - E)$ holds then the resulting class M_* of r_*-measurable sets is precisely the class $\bar{\bar{M}}$ and the restriction of r_* to M_* is identical with $\bar{\bar{r}}$.

B. V. Singbal: *A generalization of Morse relations*

Let M be an n-dimensional compact differentiable manifold of class C^3. Let f be a real-valued function of class C^2 defined on M. A point $x = (x_1, \ldots, x_n) \in M$ is a *stationary point* of f if $\frac{\partial f}{\partial x_i}$ 0, $i = 1, \ldots, n$. For a stationary point x, if $f(x) = c$ then c is a *critical level*. Let $R^k(M_1, N_1)$ be the kth Betti number of M_1 modulo N_1, where M_1 and N_1 are subsets of M. Define $f \leqslant c = \left\{ x \in M \mid f(x) \leqslant c \right\}$, $f < c = \left\{ x \in M \mid f(x) < c \right\}$. The Morse type number at the critical level c is then defined to be $m^k = R^k(f \leqslant c, f < c)$. Assuming that there are only a finite number of critical levels for which m^k is non-zero for each k, let $M^k = \sum_c m^k$. Then between M^k and R^k, the kth Betti number of M, the following relations hold : *(a)* $\sum_{r=0}^{k} (-1)^r M^{k-r} \geqslant \sum_{i=0}^{k} (-1)^r R^{k-r}, k = 0, \ldots, n$, and equality holds for $k = n$. From this also follows the inequality *(b)* $M^k \geqslant R^k$. Relation *(b)* has been generalized by Seifert-Threlfall for continuous functions defined on arbitrary topological spaces restricted by an appropriate axiom. However, relation *(a)* does not follow from *(b)*. It is shown here that by the addition of another suitable axiom, relation *(a)* also holds in the general situation. The axiom is : the critical levels are isolated and there are only a finite number of critical levels at which m^k is non-zero for each k.

APPLIED MATHEMATICS

R. D. Bhargava: *Methods of solving the torsion problem in the theory of elasticity*

If a prismatical bar is acted upon by couples whose axes lie along the axis of the bar, then an important problem of twisting the bar about its axis arises in the theory of elasticity. This is called the Torsion problem. The problem essentially has been reduced to two dimensional problems of Dirichlet and Newmann. If a plain harmonic function ϕ or its conjugate ψ can be found such that at the boundary either $(d\phi)/dr = y \cos (x, v) - x \cos (y, v)$ with v as the outward drawn normal or $\psi = \frac{1}{2}(x^2 + y^2) +$ const. then the problem is solved. The method widely used by St. Venant, is to take harmonic functions involving some constants and equating them to $\frac{1}{2}(x^2 + y^2)$, we get the cross-section for which the assumed function is a solution. Circular, elliptical and equilateral triangular sections have been solved by this method. The method of Fourier Series is utilized for rectangular or isosceles triangular sections. The cross-sections in the form of curvilinear rectangular and sector of circle have been solved if on the boundary ξ or η is constant where $\xi + i\eta = f(x + iy)$.

Among the modern methods is to map the given boundary by a unit circle in the ζ-plane. This method, though solves a wider range of problem, is not useful when the mapping function is mathematically difficult to find or when it cannot be put in the form $z = w(\zeta)$. The function space methods have also been developed by a number of workers. When the problem is mathematically intractable, a recourse to approximate method is taken. The analogy between a stretched membrane and the torsion problem introduced by L. Prandtl, is very useful. The energy method, the soap film method and the relaxation methods are the other methods which give fairly accurate results for arbitrary boundaries.

Pyare Lal and P. L. Bhatnagar: *Non-adiabatic pulsations of a stellar model*

A number of workers, Eddington, Schwarzchild, Woltzer and others, have discussed the non-adiabatic equation in connection with the problem of phase lag and maintenance and destruction of pulsations in the case of cepheid variable stars but none has given a general solution of the non-adiabatic equation. Rosseland in connection with the problem of secular stability of variable stars, has suggested a systematic method for the solution of the non-adiabatic equation. We have applied his method and obtained solution of the non-adiabatic pulsation equation in terms of a series of characteristic functions of the corresponding adiabatic equation. We have worked up to higher approximations than the previous workers. In the second part of the paper we have applied the general solution to a homogenous star, and have obtained the period of pulsation in the fundamental mode and the time of relaxation.

GEOMETRY

A. R. Rao: *A note on the Euler-line*

This note gives an invariant specification of the Euler-line of a triangle, namely the line joining the circumcentre and centroid, and a proof is given of Zeeman's theorem on the symmetrical relation among the directions of the sides and the Euler line of the triangle.

M. K. Singal and Ram Behari: *Generalizations of Codazzi's equations in a sub-space*

In a joint paper Ram Behari and R. S. Mishra [*Proc. Nat. Ins. Sci.*, (1949) *pp.* 85-92] have obtained expressions for $\nu_a \equiv x^i \lambda^i{}_{,a}$ where x^i and λ^i are respectively the direction-consines of the normal to the surface at the point x^i and the ray of the congruence through x^i. It has also been shown that if the congruence be one of normals to the surface of reference these equations reduce to Mainardi-Codazzi equations. R. S. Mishra (1952) has obtained generalized expressions for these invariants in the case of a congruence of curves, one of which passes through each point of a hypersurface V_n of a Riemannian manifold V_{n+1}.

The object of this paper is to obtain generalised expressions for these invariants in the case of a sub-space V_n of a Riemannian manifold V_m.

M. K. Singal and Ram Behari: *Some properties of a totally geodesic sub-space*

Ricci [Atti del Reale Inst. Veneto, (1904)., pp. 1233-39) has established that if a V_{n+1} admits a hypersurface with indeterminate lines of curvature for which $\Omega = 0$, (i. e. for a totally geodesic hypersurface) the normal to V_n at a point is a Ricci principal direction for V_{n+1} at points of V_n. The object of this paper is to obtain a generalization of this result for a sub-space V_n imbedded in a Riemannian V_m.

It is also proposed to show if a V_{n+1} admits ∞^1 totally geodesic hypersurfaces V_n, the tangential directions at any point are Ricci principal directions for V_{n+1} if and only if all the hypersurfaces are Einstein spaces.

K. K. Gorowara: *On Laguerre function*

The object of this paper is to obtain a new expression for the well-known Laguerre function in Differential Geometry. Various interesting properties of ruled surfaces have been obtained with the help of this function using dualistic vectorial methods.

K. K. Gorowara: *Properties of some special ruled surfaces*

The object of this paper is to obtain some properties of the two types of ruled surfaces namely: (1) Ruled surfaces generated by the edges of the trihedron of Blaschke $(A_1^{(1)}, A_2^{(2)}; A_3^{(3)})$; (2) Ruled surfaces generated by the edges of a geodesic trihedron.

STATISTICS

V. S. Huzurbazar : *On the certainty of an inductive inference*

In this paper, an interesting proposition given by Jeffreys is extended, and it is shown that repeated verifications of the consequences of a hypothesis with non-zero prior probability will make it almost certain that any number of further consequences of it will be verified. This explains a curious point arising in the Bayes-Laplace theory of sampling from an infinite (or large) population.

TOPOLOGY

M. Venkataraman : *The continuous map of a topological space into another*

In this note, we characterise the totality of all continuous real functions defined over a topological space as being a family which is functionally closed, i.e. contains with any class of functions f_a also the function $\phi(f_a)$, where $\phi(x_a)$ is a continuous function of all the arguments x_a. Analogous results hold for the totality of continuous maps of a topological space into another. The functionally closed families of real functions defined over a set R constitute a complete lattice which is lattice-isomorphic to the lattice of all complete regular topologies on R.

V. S. Krishnan and S. Swaminathan : *On completing a uniconvergence space*

A uniconvergence in a space has been defined (V. S. Krishnan, *J. Madras Univ.* B., XXIII, 2, pp. 174-181, 1953) as a scheme of convergence of point sets indexed by a fixed directed set D, these convergent directed sets being subject to certain axioms. The structure obtained is similar to the uniform structure defined by A. Weil. And one can pass from one of these two structures to the other in both directions. Here we study the process of completing a uniconvergence space.

The Cauchy directed set and a star convergent set of points, both indexed still by D, are suitably defined and completeness is defined to mean that every Cauchy directed set is star convergent. Then the completeness of a uniconvergence space is shown to imply and be implied by the completeness under the associated uniform structure. A completion $\tilde{R}$ of R obtained by adjoining all Cauchy directed sets of R is then subjected to an identification E of certain subsets, but not of the distinct points of the original space R (considered as immersed in $\tilde{R}$). This completed space considered as a uniconvergence space is in one-to-one correspondence with a subset S of the completion R^* of the original space R considered as a uniform space, and can be a proper subset in particular cases. The identity transformation is uniformly continuous from S with the uniformity deduced from R/E into R^* with its uniformity. When R itself is a Hausdorff space, an identification of topologically equivalent points in $\tilde{R}$ or in R^* is possible without identifying distinct points of the subspace R. And these give a complete uniconvergence space and a complete uniform space respectively, which are isomorphic (i.e. there is one-to-one correspondence between these under which the uniconvergence and uniformity are deducible one from the other).

MATHEMATICAL EXHIBITION

GENERAL

1. Group Photographs of the last eighteen conferences of the Indian Mathematical Society

1st	1916	Madras	2nd	1919	Bombay
3rd	1921	Lahore	4th	1924	Poona
5th	1926	Bangalore	6th	1928	Nagpur
7th	1931	Trivandrum	8th	1932	Bombay
9th	1935	Delhi	10th	1937	Lucknow
11th	1939	Hyderabad	12th	1941	Aligarh
13th	1943	Annamalainagar	14th	1945	Delhi
15th	1947	Waltair	16th	1949	Madras
17th	1951	Bangalore	18th	1952	Bombay
19th	1953	Delhi			

2. Journal of the Indian Mathematical Society

Vol. I.—Vol. XX	1909-1933	Complete Sets.
New Series Vol. I.—Vol. XVIII	1934-1953	

3. Mathematics Student Vol. I—Vol. XX 1933-1953 Complete Set.
4. Tree of Knowledge.
5. Chart showing relationship between various branches of natural science and technology.
6. A very brief history of mathematics.

Puzzles, Paradoxes, and Mathematical Recreations

1. The Problem of Eight Engines
2. The Problem of separating the two pieces of a box
3. The Problem of Three Dice
4. The Problem of interchanging the two Bogies by an Engine
5. The Problem of the four mice
6. The " T " jig-saw puzzle
7. Letters obtained by cutting a chess board forming the sentence " Cut Thy Life "
8. Making of two elliptical tops of two stools from a circular top of a table
9. The Problem of " Mutual Suspicion " with solution
10. The Ring Thief
11. " Are you still unmarried ? " If so, learn Integration
12. " Choose any Number "
13. " The Four Sevens "
14. " Look and Look Again "
15. " Count the Squares "
16. The Circle Problems

17. On the Chess Board
18. Fill in the gaps, with solution
19. Torpedo Practice, with solution
20. " Tangrams "
21. The Wall Problem
22. Paradoxes of Zeno
23. St. Lines only
24. $\pi = 2$
25. $\sqrt{a^2 + b^2} = a + b$
26. M.A. = B.A.
27. Number curiosities
28. Ornamental Squares
29. Fun with numbers
30. Think of a number
31. Window-Reader
32. Curves of constant breadth
33. Count the triangles
34. Is mathematics not interesting ?

Hindu Mathematics

1. Cube Root
2. Magic square 8×8
3. Rashi Chakra
4. Sarvato Bhadra
5. Magic squares with total zero
6. Superstition
7. 18×18 magic squares
8. Square root
9. Cubing
10. Narayan squares
11. Squaring
12. Key-Square
13. Magic square total 175
14. 18×18 Magic square total 2925
15. Magic circle
16. Dvadeskar
17. Magic Knights' tour
18. Burmese chess-board
19. Mandap

Paper Models of Polyhedra

1. Tetrahedron 3^3
2. Octahedron 3^4
3. Icosahedron 3^5
4. Cube 4^3
5. Truncated tetrahedron 3.6^2

6. Dodecahedron 5^3
7. Cuboctahedron $(3.4)^2$
8. Truncated cube 3.8^2
9. Truncated octahedron 4.6^2
10. Rhombicuboctahedron 3.4^3
11. Truncated cuboctahedron 4.6.8
12. Snub cube $3^4.4$
13. Icosidodecahedron $(3 \cdot 5)^2$
14. Truncated dodecahedron 3.10^2
15. Truncated icosahedron 5.6^2
16. Snub dodecahedron $3^4.5$
17. Rhombicosidodecahedron 3.4.5.4
18. Rhombic triacontahedron $(3.5)^2$

(19-28). 10 other paper models of duals.

GEOMETRY

Plaster Models

1. Hyperboloid of two sheets
2. Elliptic paraboloid
3. Intersection of cylinder and hyperbolic paraboloid giving lines of curvature
4. Hyperbolic paraboloid giving sections by co-ordinates planes
5. Ellipsoid with lines of curvature
6. Helicoid (a surface of constant negative curvature)
7. A particular case of Dupin's Theorem. Intersection of a triply orthogonal system of surfaces showing " lines of curvature of an ellipsoid are its lines of intersection with the confocals "
8. Prolate spheriod with particular geodesics (with a seat)
9. Ellipsoid giving sections parallel to one of the principal planes
10. Ellipsoid giving principal sections
11. Hyperboloid of two sheets giving section parallel to one of the principal planes
12. Elliptic paraboloid showing sections parallel to one of the principal planes
13. Catenoid, a surface of constant-negative curvature, with geodesics
14. Elliptic cone
15. Surface obtained by revolving a tractrix giving geodesics and lines of curvature
16. A surface of positive curvature (Helocoid wound on a cylinder)
17. Helicoidal screw
18. Projections of plane cubics on a sphere
19. Projections of plane cubics on a sphere
20. Riemann Surface 6 $w = e^{1/z}$
21. Riemann Surface $w^4 = 1 - z^2$
22. Intersection of quadrics giving rise to space curves
23. A quartic surface
24. Surface of revolution obtained by revolving a cycloid
25. Riemann surface
26. Wave surface

27. Cyclide
28. Surface generator by an ∞ of spheres
29. Bicircular quartic surface
30. A Riemann surface
31. A quartic surface generator by $a\infty$ of sphere
32. An extremely oblate spheroid

Wooden Models

1. A complete conical boring in a sphere
2. A partial conical boring in a sphere
3. A cylindrical boring in a cylinder (two pieces)
4. Cylindrical boring in a sphere
5. Elliptic section of a right cone
6. A complete conical boring in a cylinder
7. Caps of a sphere (three pieces)
8. Rectangular pyramidal giving its frustrum and volume (in ten pieces)
9. Model expressing the relations : area of a circle = area of a rectangle contained by πr and r (two pieces)
10. Model showing volume of a pyramid = 1/3 that of a right prism on the same base
11. Frustrum of a right prism on a polygonal base
12. Pyramids on square base (two pieces)
13. Volume of a right prism etc. (two pieces)
14. Model explaining the conic sections : ellipse, parabola and hyperbola (4 pieces)
15. Parallelopiped

Wooden Models

1. Ellipse with quadrants
2. Rhombus
3. Triangle
4. Isosceles triangle (two pieces joined together)
5. Rectangle (four pieces joined together)
6. Equilateral triangle (six pieces all joined together)
7. Square (four pieces all joined together)
8. Parallelogram (coloured : 4 pieces, all joined together)
9. Rhombus (coloured) : divided into four triangles
10. $2(a^2 + b^2) = (a + b)^2 + (a - b)^2$
11. Theorem of Pythogoras
12. Congruency of right-angled triangles
13. Cube in two pieces
14. Theorem of Pythogoras (five pieces)
15. $(a + b)^2 - (a - b)^2 = 4\,ab$
16. Area of triangle = $\frac{1}{2}$ area of rectangle with the same base and the same height
17. Plane sections of (i) cone (ii) cylinder (iii) pyramid
18. Universal joint
19. Hyperboloid of one sheet (wood & thread)

Tin Models

1. Explaining the generators of hyperboloid of one sheet
2. Octants of sphere, eight co-ordinate planes (each containing four pieces)
3. A cone in two pieces giving hyperbola as the sections
4. A triangular pyramid
5. A frustum of a triangular pyramid
6. Giving total surface of a right cone
7. A spherical honey-comb with twenty houses also explaining spherical triangles (five pieces)
8. Ellipse
9. Parabola
10. Hyperbola
11. A Hemispherical bowl
12. A cylinder with a cone on the same base and height equal to the radius of the bowl (two pieces)

Paper and Card-board Models

1. Hyperbolic parabolid
2. Area of a triangle = $\frac{1}{2}$ area of a rectangle on the same base and on the same height
3. Ellipsoid showing circular sections
4. Hyperboloid of one sheet (cardboard & thread)

Wire Models

1. Octahedron
2. Eight-faced polyhedra, each face being equilateral triangle

3-7. 5 wire models showing crystal zones

8. Hyperbolic paraboloid
9. Hyperboloid of one sheet
10. Hyperboloid of two sheets
11. Desargue's configuration

PURE MHTHEMATICS

1. Ten commandments (4 charts)
2. Value of π to 810 places
3. Value of e to 2500 places
4. 12 Perfect numbers
5. Optical illusions (9 in numbers)
6. Cissoid of Diocles and its application to duplication of cube
7. Perpetual Calendars (5)
8. Hindu Calendar (1)
9. Do you Know ? (5 charts) 32 results

10. Arithmetic of Gaussian Integers (chart)
11. Secret Messages (chart)
12. Some famous problems (14 problems ; one chart)
13. Klein Bottle (blown from glass)
14. Torus divided into seven regions each touching the other six
15. Möbius surfaces of 1st, 2nd and 3rd order and surfaces obtained from these by cutting in different ways
16. Soap film experiments

ASTRONOMICAL (CHARTS AND PHOTOGRAPHS)

1. (i) Representation of constellations of nothern regions of the sky. (ii) Perseus and Andromeda (iii) Orion (iv) Perseus and Auriga (v) Chart showing stars of varying brightness.

2. Signs of the Zodiac.

3. (i) Inter-planetary distances in terms of speed of a rocket-ship.
   (ii) Astronomical distances in terms of velocity of light.
   (iii) Relative sizes of sun and planets.
   (iv) Relative masses of sun and planets.
   (v) Saturn and its rings.
   (vi) Trail of a meteor.
   (vii) Moon.
   (viii) Development of comet Cunningham.
   (ix) Orbit of a comet.
   (x) Travel of a minor planet
   (xi) Orbits of Apollo, Adonis and Hermes.

4. Solar system : A chart giving symbol, mean distances, mass, period, atmosphere, temperature, eccentricity and inclination of the orbit, mean orbital velocity diameter, satellites, date of discovery, sidereal period of the sun and the planets.

5. Origin of Solar system : (i) Kant's theory of rotating mass (Nebulae) (ii) Laplace's Theory of rotating nebulae of gases (iii) Chamberlin—Moulton Theory (iv) Jeans-Jeffery's Theory (v) Berlage's Theory (iv) Cytellton's Theory (vii) Alfven's Theory (viii) Hoyle's Theory (ix) Weizsaecker—Schmidt Hypothesis of Vortices.

6. (i) Harvard Meteor Statistics (ii) A Meteor Trail (iii) Super-Schmidt Camera for Meteor Study (iv) Solar Corona.

7. (i) Fraunhoffer Spectra of the sun (ii) Spicules above the sun (iii) Spectrum of the chromosphere. (iv) Spectra of light of Hydrogen and ionised calcium (v) Corona.

8. (i) Photographs of the Sun's disc taken at Kodaikanal (ii) Sun-spots (iii) 11 year cycle of the Sun-spots.

9. Sun-spot group of March, 1947.

10. (i) Spectroheliograph building (ii) Foucault Siderostat (iii) Cooke Photo Visual Lens (iv) H.L.K. Spectroheliogram of Sun's disc (v) K—Spectroheliogram (vi) K—Spectroheliogram of Solar limb (both of the quiet and the active Sun) (vii) Eruptive prominences (viii) Solar flare (ix) K—Spectropheliograms of Sun's disc on 7-2-44 and 7-4-47.

11. Solar Prominences (8 photographs).

12. Solar Prominences (10 photographs).

13. (i) Solar Prominences (8 Photographs) (ii) Solar Flares (2 photographs) Chart of Carbon-Nitrogen cycle (iv) Composition of elements in earth and sun.

14. Chart showing 20 brightest stars, 10 nearest stars, 10 brightest double stars, 10 bright variable stars, star-diameters and surface temperature also giving some data about them.

15. (i) Cross-section of a galaxy (ii) Two diagrams showing classification of galaxies (iii) Diagrams showing steller population and lines of equal mass and equal radius (iv) Two spectra photographs.

16. (i) Section of Milky way (ii) Nebula (iii) Magellanic Clouds.

17. (i) Milky way 2 photographs (ii) Omega-Centauri—2 photographs (iii) Globular Cluster M 13 (iv) Birght Nebulae in Prism Constellation (v) Orion Nebulae (vi) 4 more photographs of various nebulae.

18. (i) Horsehead nebulae (ii) Appearance and disappearance of Nova (iii) Nova Pictoris.

19. (*a*) The Hindu Ecliptic and northern yogatārs with modern names—charts.

(*b*) Astronomical instruments-photographs.

(i) Dome of 200″ Hale telescope. (ii) 82″ Telescope at McDonald (iii) Armagh-Dunsink-Harvard (3″-36″) Schmidt telescope. (iv) Telescope at Yearkes observatory (v) Spectroheliograph at Kodaikanal—2. (vi) Ohio state University Radi telescope (vii) Artist's drawing of the largest parabolic antena for a radio telescope (viii) V-2 Rocket.

(*c*) Instruments at Kodaikanal.

(i) 12″ Coelostat (ii) View of observatory (iii) Spectrohelioscope (iv) Littrow prism spectrograph (v) 10′ concave grating spectrograph (vi) Solar spectrum (vii) Sun spot spectra (viii) Solar Spectra over a sun spot (ix) Polar Siderostat (b) 12″ Siderostat (c) 6″ Coelostat.

d) Cosmic ray and photographic work at Kodaikanal.

(i) Photoelectric photometer (ii) Microphotometer (iii) Sky Photometer (iv) Hut for cosmic ray recorder (v) Record of density variation (vi) Record of intensity variations.

(*e*) Astrowork at Kodaikanal.

(i) Transit House (ii) Dome for Grubb reflector (iii) 6" Transit instrument. (iv) 20" Grubb reflector (v) Dome for Cookes reflector (vi) Cookes telescope.

(*f*) Magnetic work at Kodaikanal

(i) Kew Magnetometer (ii) Magnetic observetory (iii) H. F. Magnetograph (iv) V. F. Magnetograph (v) D-Magnetograph (vi) Magnetic storm (vii) Normal diurnal variation

(*g*) Ionospheric work at Kodaikanal.

(i) Library with extension in progress (ii) Ionospheric Laboratory (iii) Shelton Clock (iv) Ionosphere recorder (v) Ionospehric recorder.

(*h*) Nebulae.

(i) Andromeda Nebulae (ii) Andromeda Nebula (iii) Sprial Galaxy (iv) Cluster of Galaxies (v) Nebulae.

(*i*) Nebulae.

(*j*) Nebulae

(i) Bright Nebula (ii) Bright Nebulosity in Carina (iii) Enlargement of previous picture (iv) Dark Nebulae in Milky way (v) Spiral Galaxy M51 (vi) Spiral Galaxy M81 (vii) Spiral Galaxy M101 (viii) Spiral Galaxy M31 (ix) Cone Nebulae (x) Coal Sack Nebula.

(*k*) Photographs of astronomers.

(i) H. N. Russell (ii) J. H. Jeans (iii) M. N. Saha (iv) Eddington (v) Shapley (vi) Abetti (vii) Minaert and Thackrey (viii) Otto Struve (ix) Paul Merill and Oort (x) Lemaitre (xi) Lyot (xii) Lindblad, Spencer Jones, Oort (xiii) Waldmeier

(*l*) External Galaxies

(i) Elliptical Galaxies (ii) Spiral Galaxies (iii) Large and small Magellanic Clouds (iv) Andromeda Nebula (v) Spiral Galaxy in Ursa Major (vi) Spiral Galaxy M51 (vii) Spiral Galaxy M81.

24 photographs of solar prominences, sun spots etc.

PHYSICS

1. Chart of Electromagnetic Radiations
2. Chart of Known Dimensions
3. Normalised wave functions of linear oscillator
4. Experiments to illustrate principle of Moments and to illustrate the generation of S. H. M.
5. Chemical compound model
6. Surveying Instruments 3

STATISTICS

1. Statistical quality control (3 Charts)
2. Confidence Belts for ρ (1 Chart)
3. Frequency distributions (2 Charts)
4. Confidence limits for Binomial Distributions (1 Chart)
5. Bivariate Histogram (1 Chart)
6. χ^2 approaches normality (1 Chart)
7. χ^2 probability curves (1 Chart)

BALLISTICS

1. Shape of a projectile (3 charts and 7 photographs)
2. Trajectory of a projectile in air (2 charts)
3. Resistance laws (3 charts)
4. Hollow charge (6 photographs)
5. Variation in maximum pressure, muzzle velocity, All-Burnt position with loading conditions (charge wt., propellant size, shape, chamber capacity, shot-start pressure, bore area (7 photographs)

HYDRODYNAMICS AND AERODYNAMICS (CHARTS)

1. Principle of Wind tunnel Balance
2. Terminal velocities
3. Carburettor
4. Wind Tunnel with experimental section down stream (From the Fan)
5. Wind Tunnel of the Effel Type
6. Wind Tunnel with continuous closed circuit
7. Pressure distribution over an aerofoil
8. The Approach and Landing
9. Lift-Drag Curve
10. Lift-Curve
11. Drag-Curve
12. Speed square law
13. Change in temperature, pressure and density of air with height
14. Types of Aircraft

MEMBERS OF THE CONFERENCE

DELEGATES

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M. S. Mukheja, V. V. R. Murthy, M. H. Musavi, Nagendra, Niamat Rai, Miss Sarojini Pandit, Peary Lal, N. Prakash, B. Prasad, T. R. Puri, R. N. Rai, D. Raja Ram, Rajinder Singh, B. Rama Murti, Ram Behari, Ram Lal, S. R. Ranganathan, H. L. Rishi, Sachdeo Singh, N. K. Saha, Sahib Singh, P. Saran, H. C. Saxena, S. C. Sen, G. R. Seth, G. R. Sethi, S. C. R. Shah, Shamsher Bahadur, Shanti Narayan, T. R. Seshadri, V. Shibayev, P. D. Shukla, M. K. Singhal, S. C. Sircar, S. P. Singh Sodhi, K. N. Sreenivasa, H. S. Subba Rao, S. Subramaniam, R. N. Sundaram, Swami Jnananda, Mrs. S. V. Tawadey, Thakur Singh, Miss V. Thakurdas, Miss C. Thiruvady, S. Vadantam, R. S. Varma, Ved Prakash, Ved Prakash, N. S. Venkatesan, P. D. Verma, Vidya Ratna, Nanak Chand Vij, Virendra Kishore, Prabody Vyas.

NEWS AND NOTICES

The following members have been admitted to life-membership in the Society : S. Chowla, V. S. Krishnamurti, N. M. Shah, and L. N. Subramanian.

The following persons have been elected ordinary members of the Society : D. N. Abrol, H. R. Aggarwal, V. Goswami, B. S. Grewal, Hukum Chand, S. R. Khanwalkar, S. S. Mathur, P. V. Patel, and A. R. Rao.

The Council of the Society for the session 1953-1955 is as follows : Professors Ram Behari (President), S. Mahadevan (Secretary), C. N. Srinivasiengar (Treasurer), K. Chandrasekharan (Editor), T. Vijayaraghavan (Librarian), P. L. Bhatnagar, G. L. Chandratreya, Hansraj Gupta, B. S. Madhava Rao, S. Minakshisundaram, N. S. Nagendra Nath, A. Narasinga Rao, B. N. Prasad, N. G. Shabde, and A. N. Singh.

At a meeting of the Council held in Delhi on December 25, 1953, the terms of award of the Narasinga Rao Medal were determined. It was decided that a gold medal of the value of Rs. 100/- be awarded every year to a person of Indian domicile, under thirty years of age, for the best paper appearing either in the *Journal* or the *Mathematics Student* for that year, on the strength of a unanimous recommendation made by a committee consisting of the President, the Editor, and a nominee of the Council. No candidate will be eligible for the medal more than once.

Dr. T. Vijayaraghavan is the nominee of the Council, for the current year, on the Narasinga Rao Medal Committee.

A Directory is proposed to be prepared of all members of the Society. Members are requested to send in correct details regarding their present address, academic qualifications, academic position, etc. to Professor S. Mahadevan, 2 Bhaskarapuram, Mylapore, Madras-4.

The University of Madras has furnished the National Committee for Mathematics with copies of the unpublished manuscripts of the

late Srinivasa Ramanujan. The National Committee is taking steps to secure the services of experts to examine the manuscripts and to advise further action.

The Madras Mathematics Club has elected Professor S. Mahadevan (President) and Mr. S. Swaminathan (Secretary) as office-bearers for 1954-55.

On the retirement of Professor N. M. Basu, Dr. S. M. Shah has been appointed Professor and Head of the Department of Mathematics, Muslim University, Aligarh.

Dr. H. C. Gupta, Lecturer, Muslim University, Aligarh, has been awarded a Government of India Research Scholarship and has joined the University of Manchester.

Dr. Jamil Ahmed Siddiqui, Lecturer, Muslim University, Aligarh, has received the D.Sc. degree of the University of Paris.

Dr. U. N. Singh of the University of Allahabad has received the D. Sc. degree of the University of Paris.

Mr. S. K. Singh of D. S. College, Aligarh, has received the Ph.D. degree of the Muslim University, Aligarh.

The Royal Astronomical Society of Great Britain has awarded a gold medal to Professor Subrahmanya Chandrasekhar of the University of Chicago.

International Congress of Mathematicians 1954

Second Communication

The Organizing Committee is pleased to inform you of the following details. The Committee regrets that for technical and administrative reasons this second communication could only be written in one language, although the Committee would have liked to use more languages. For similar reasons the task of the Secretariat would be lightened very much if in your correspondence with the Congress Committee you would use English, French or German.

General

Date and place. The Congress will be held in Amsterdam from Thursday September 2nd to Thursday September 9th (inclusive) and will meet in the building of the Royal Tropical Institute. The opening and closing sessions will be held in the Amsterdam " Concertgebouw."

Secretariat. All correspondence must be directed to *The Secretariat of the International Congress of Mathematicians* 1954, 2e *Boerhaavestraat* 49, *Amsterdam, The Netherlands.*

Membership. As stated in the first communication there will be two categories of members in the Congress : *regular members,* paying D. Gld. 50,—(or, if they do not want to attend the banquet, D. Gld. 40,—) and *associate members,* paying D.Gld. 20,— (or, if they do not want to attend the banquet D. Gld. 10,—).

Finance. The fees may be paid immediately (and preferably before Febr. 15th, 1954) in the account of the International Congress of Mathematicians 1954 with the " *Amsterdamsche Bank N.V., Filiaal LEIDEN, The Netherlands.*" Those intending to pay by postal transfer, may use the "Postgiro" number of this Bank, Filiaal LEIDEN, No. 9200.

Scientific Program

One-hour-addresses : Until now a score of outstanding mathematicians have been invited by the Organizing Committee to deliver one-hour-addresses ; most of them have accepted the invitation. The Organizing Committee is convinced that by these addresses a survey of the recent development in the whole field of mathematics will be furnished.

Sections. As stated in the first communication there will be seven sections.

Half-hour-addresses: Most of the 45 experts who, up to the present, have been invited to deliver half-hour-addresses have accepted

the invitation. The distribution over the Sections is intended as follows :

Section	I	Algebra and Theory of Numbers	(8 addresses)
,,	II	Analysis	(13 ,,)
,,	III	Geometry and Topology	(8 ,,)
,,	IV	Probability and Statistics	(4 ,,)
,,	V	Mathematical Physics and Applied Mathematics	(6 ,,)
,,	VI	Logic and Foundations	(3 ,,)
,,	VII	Philosophy, History and Education	(3 ,,)

Short lectures : Short lectures will be given by regular members of the Congress who apply beforehand to the Organizing Committee. The time allotted for a short lecture is 15 minutes. The Organizing Committee has the intention to give copies of the collected preprints of these lectures to all regular members at the beginning of the Congress. It is therefore essential that the Organizing Committee should be in possession of the abstracts of the papers concerned before February 15th, 1954 ; Address : *Secretariat of the International Congress* 1954, 2e *Boerhaavestraat* 49, *Amsterdam*, *The Netherlands*. These abstracts should be made on the blank enclosed herewith; the text will be considered final. The abstract may not exceed 400 words. It is by no means sure that applications which reach the Organizing Committee after February 15th, 1954 can be accepted.

Proceedings: The Proceedings will be printed after the Congress. Each regular member will receive the Proceedings without further payment. Further copies may be available at a reduced price for those who express their wish before March 15th 1954. As till now no estimate could be made as to the size of the Proceedings, the price could not be fixed yet.

Entertainments and Excursions

The Organizing Committee has planned several entertainments including official and informal receptions and parties, a concert by the " Concertgebouworkest Amsterdam " and a banquet. At these

features no special dress is required. Moreover a number of excursions have been planned which may give an impression of the landscape and the culture in the Netherlands. A special ladies-program has been compiled. In view of the preparation of some of these recreations the Organizing Committee invites you to complete the prescribed form, in order to know your preference concerning :

A. *A number of excursions by bus on Tuesday-afternoon, Sept. 7th,* 1954 :—

- A 1 Amsterdam—Velsen (Blast-furnaces and Steel Works) —IJmuiden (Northsea-sluices) (4 hours).
- A 2 Amsterdam—Amsterdamse Bos (park of 1000 ha.)—Schiphol Airport—Cruquius (hydraulic model of the Dutch polders)—Haarlem—Amsterdam (4 hours).
- A 3 Amsterdam—Volendam—the Island of Marken (characteristic houses and costumes)—Amsterdam (4 hours).
- A 4 Amsterdam—The Hague (Museums, Peace-Palace)—Scheveningen (beach)—Amsterdam (5 hours).
- A 5 A trip through Dutch polders to the artificial dike between Holland and Friesland, intended for those who are interested in the specific hydraulic problems of Holland (6 hours).
- A 6 Amsterdam—Delft (historical town, cathedral)—The Hague—Amsterdam (6 hours).
- A 7 Amsterdam—Rotterdam—Amsterdam (6 hours.)
- A 8 A trip through the polders to the old Zuiderzee-ports Hoorn, Enkhuizen (6 hours).

B. *A number of evening-entertainments which are planned for Monday-evening, September 6th,* 1954 :

- B 1 Chamber music by a Dutch stringquartet.
- B 2 Chansons-evening.
- B 3 Show of historical costumes.
- B 4 Simultaneous chess-game by Dr. M. Euwe (mathematician and former world champion).
- B 5 A programme of Dutch cultural films.

C. *A number of morning-excursions which are planned for September 3rd, 6th and 8th and which are intended for the associate members :*

- C 1 A flower auction (Aalsmeer).
- C 2 An old Dutch castle (Muiderslot).
- C 3 Haarlem (Frans Hals museum and old cathedral).
- C 4 Typical old Dutch villages.
- C 5 Diamond Factory.
- C 6 Museum in Amsterdam.
- C 7 Fashion show.
- C 8 Schiphol Airport.
- C 9 A special visit to a department store.

Hotel Accommodation

The Organizing Committee has reserved a number of hotelrooms and a number of rooms with private families. As during summer and autumn Amsterdam is overcrowded by tourists, it is essential that all those who intend to participate in the Congress (including those who prefer to arrange their own lodgings) complete the prescribed form.

Prices of hotelrooms. Hotelrooms may be grouped in the following price-classes (including bed and continental breakfast but no further meals) per day and per person :

A. D. Gld. 15.—to D. Gld. 18.—
B. D. Gld. 9.—to D. Gld. 14.—
C. D. Gld. 4.—to D. Gld. 8.—

Category C includes not only hotels, but also "pensions" (boarding houses). All prices mentioned are subject to the addition of a 12—15 % service charge. Finally the Organizing Committee is trying to find inexpensive accommodation in private houses or in youth-hostels, the latter for those participants (e.g. younger single people), who have no objection to share a dormitory with a number of other Congressists. The price of such lodgings may vary between D. Gld. 1.50 and D.Gld. 3.50.

As the demand for rooms of all kind will probably be greater than the number of such rooms available, we advise you in your own interest, to send in your Hotel Booking Form as soon as possible (and preferably before Febr. 15th, 1954), as the application will be dealt with according to the date of receipt by the Organizing Committe. If the number of participants exceed a certain limit, the Organizing Committee may be obliged to make use of hotel accommodation in the neighbourhood of Amsterdam, in which case the Organizing Committee will arrange transport. The Organizing Committee reserves the right to combine two orders for single rooms into one single order for a double room on account of the limited number of single rooms available. For the same reason the Organizing Committee reserves the right to cancel an order for a bathroom.

Travelling Accommodation

The Organizing Committee has contacted the Holland-America-Line to make a special arrangement for the American participants for accommodation in the following sailings :

New-York—Rotterdam :

s.s.	" Ryndam "	dep.	July	28th	arr.	Aug.	6th	1954
m.s.	"Westerdam"	,,	Aug.	1st	,,	,,	9th	,,
s.s.	"Maasdam"	,,	,,	5th	,,	,,	14th	,,
m.s.	"Noordam"	,,	,,	14th	,,	,,	23rd	,,
s.s.	"N. Amsterdam"	,,	,,	17th	,,	,,	25th	,,
s.s.	"Ryndam"	,,	,,	21st	,,	,,	30th	,,

Rotterdam—New-York :

m.s.	"Westerdam"	dep.	Sept.	11th	arr.	Sept.	20th	1954
s.s.	"N. Amsterdam"	,,	,,	17th	,,	,,	25th	,,
s.s.	"Maasdam"	,,	,,	20th	,,	,,	29th	,,
m.s.	"Noordam"	,,	,,	25th	,,	Oct.	4th	,,

Further information as to conditions and further sailings will be furnished by the American Office of the abovementioned Steamship-Company : Broadway 29, New-York and also by Prof. E. G. Begle,

Secretary of the American Mathematical Society, Yale University, New Haven 11, Connecticut. It will be possible to book for a round-trip as well as for a single trip only.

A similar project is planned by the Royal Dutch Airlines (K.L.M.); information will be given by the Headquarters of the K.L.M. in your country.

Rate of exchange : At the moment is :

1 U. S. $	=	D.	Gld.	3.80
1 £	=	D.	Gld.	10.64
1 D. M.	=	D.	Gld.	0.90
100 Belg. frs	=	D.	Gld.	7.60
100 Fr. frs	=	D.	Gld.	1.08

General advice. Participants are advised to be prepared for the changeability of the Dutch climate. Temperatures may vary from 12°—20° Celsius.

International Mathematical Union

The International Mathematical Union has already given financial support to symposia in certain fields of mathematics. In order that further such work may be carried out most effectively, the Secretary of the Union asks for proposals for future symposia to which the Union may eventually decide to offer its support. The proposals which are sent in should state the subject of the symposium, should contain an estimate of its cost, and give indication of the projected sources of support. The Executive Committee of the Union is particularly desirous of making plans for the years 1955-56. All information in this connection should be submitted, before the end of June 1954, to Professor Enrico Bompiani, Secretary of the International Mathematical Union, Institute of Mathematics, University of Rome, Rome, Italy.

THE INDIAN MATHEMATICAL SOCIETY

FOUNDED 1907

SESSION 1953-55

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The Society publishes two periodicals, *The Journal of the Indian Mathematical Society* and the *Mathematics Student*. The annual subscription for the *Journal* is rupees twelve, and that for the *Student* is rupees six. Back volumes of both the periodicals are available except for a few numbers out of stock. The following publications of the Society are also available : (i) Memoir on cubic transformations associated with a desmic system, by Dr. R. Vaidyanathaswamy, pp. 92, Rs. 3. (ii) Tables of Partitions, by Dr. Hansraj Gupta, pp. 81, Rs. 5.

EDITED BY K. CHANDRASEKHARAN,

TATA INSTITUTE OF FUNDAMENTAL RESEARCH, BOMBAY 1,

PUBLISHED BY THE INDIAN MATHEMATICAL SOCIETY, AND

PRINTED BY S. RAMU AT THE COMMERCIAL PRINTING PRESS, BOMBAY 1.

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JANUARY 1954

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